Technical Note

How does an interference filter work?

Introduction

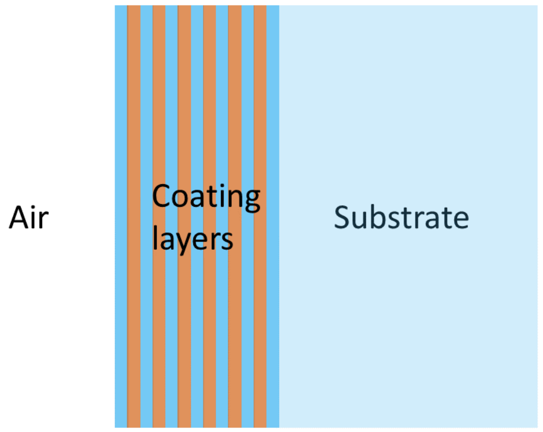

Figure 1: Interference filter consisting of a substrate with a stack of coating layers on one side.

Interference filters are very effective to accomplish almost any type of spectral optical filtering. Typical examples are short-wave pass, long-wave pass and bandpass filters. But also, more complex filters like multi-bandpass filters, continuously variable filters and dichroics can be implemented.

One of the key benefits of interference filters is, that almost any spectral filtering function can be designed and implemented. An interference filter consists of a flat substrate (often glass) with many thin coated layers of dielectric materials on either one side or both sides of the substrate. The coating layers alternates between a material with high and low index of refraction.

This technical note explains the fundamentals behind how an interference filter works.

Interface between two dielectric layers

Before investigating how the dielectric layers form an interference filter, it is useful to look at the interface between two layers.

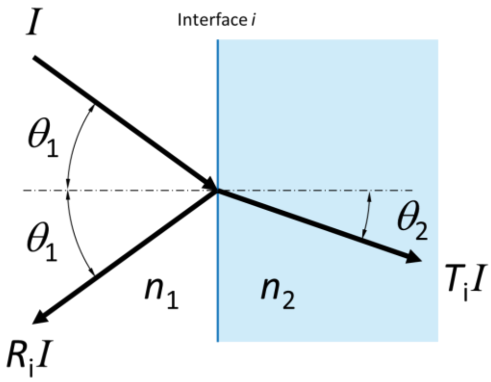

Figure 2: Reflection and transmission of the incident light intensity I at the interface i between two dielectrics with refractive indices ***n_1*** *and* ***n_2****, respectively*

When light is incident on the interface (labeled *i*), some fraction (R_i) of the light intensity is reflected and some fraction (T_i) is transmitted. If we assume that the interface is loss-less, which is a good approximation for dielectrics, the conservation of energy dictates that:

$$T_i = 1 - R_i$$

The relation between the angle of the incidence and the transmitted light relative to the normal is given by Snell's law of refraction:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

The reflection and transmission (intensity) coefficients are different for the s- and p-polarized light and depend on the refractive indices and angle of incidence as follows:

$$R_s = \left(\frac{\sin(\theta_1 - \theta_2)}{\sin(\theta_1 + \theta_2)}\right)^2$$

$$T_s = 1 - R_s$$

$$R_p = \left(\frac{\tan(\theta_1 - \theta_2)}{\tan(\theta_1 + \theta_2)}\right)^2$$

$$T_p = 1 - R_p$$

For the interface between air ($n_1 = 1$) and glass ($n_2 = 1.5$) the reflection and transmission coefficients for the s- and p-polarized light as a function of angle of incidence are shown on Figure 3.

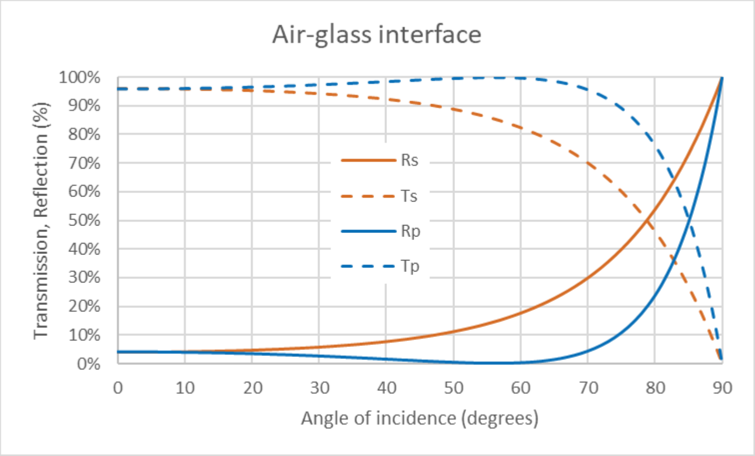

Figure 3:Reflection and Transmission coefficients for glass-air interface as a function of angle of incidence θ_1.

Single dielectric layer

Often, interference filters employ hundreds of coating layers but, to understand the fundamentals, we will consider only a single dielectric layer as show on Figure 4. The dielectric layer has a thickness *d* and a refractive index n_2 and is surrounded by a media of refractive index n_1 to the left and n_3 to the right.

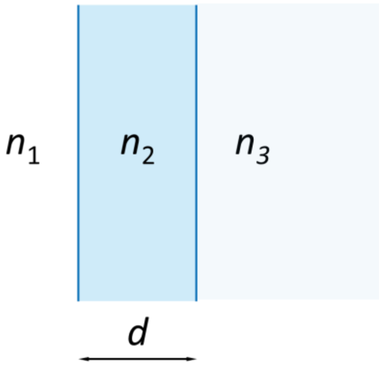

Figure 4: Dielectric layer of thickness ***d*** *and refractive index* $\boldsymbol{n_2}$ *surrounded by materials with refractive indices* $\boldsymbol{n_1}$ *and* $\boldsymbol{n_3}$ *respectively.*

When light is incident on the surface of the dielectric layer from left, some fraction of the light is transmitted through the surface and the remaining light is reflected as indicated on drawing 1 on Figure 5. Drawings 2 to 4 illustrate how the light transmitted into the dielectric layer bounces back and forth between the two surfaces and, for every bounce, some amount of light is transmitted through the surface. Figure 5 only shows the first few bounces but there is an infinite number of reflections and transmissions taking place.

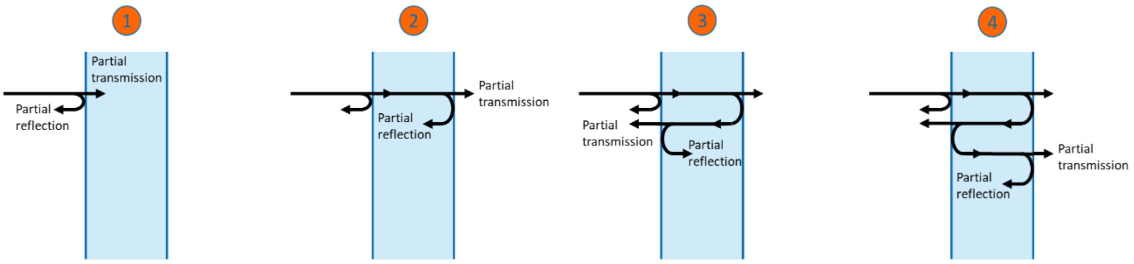

Figure 5: Illustration of multi partial reflections and transmissions taking place at the two surfaces of the dielectric layer.

On each side of the dielectric layer, we end up with a sum of transmitted (forward moving) waves and a sum of reflected (backwards moving) waves. If the forward moving waves are all in phase, as shown on Figure 6, the waves add up to produce maximum transmitted light through the dielectric layer. Consequently, the backwards moving waves will be out of phase and minimal amount of light is reflected.

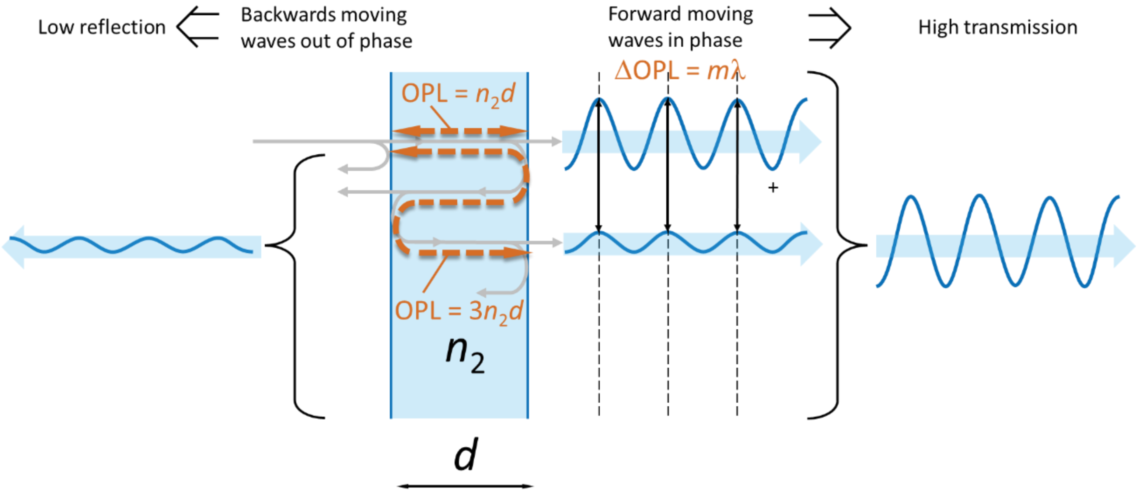

Figure 6: High transmission through the filter when the forward moving waves are all in phase; ΔOPL = mλ.

For the forward moving waves to be all in phase the difference between the **Optical Path Length (OPL)** each wave experiences when traveling through the dielectric layer should be an integer number m of wavelengths λ. This can be written as:

$$\Delta \text{OPL} = m\lambda$$

The optical path length equals the distance travelled in a medium multiplied with the refractive index of the medium. This means that the OPLs for the directly transmitted and the first double reflected wave through the dielectric layer (see Figure 6) are

$$\text{OPL(directly transmitted)} = n_2 d$$

$$\text{OPL(double reflected)} = 3n_2 d$$

and thereby

$$\Delta \text{OPL} = 2n_2 d$$

In the case where we want maximum transmission through the dielectric layer, ΔOPL should equal an integer number of wavelengths ($\Delta\text{OPL} = m\lambda$) which means that for maximum transmission the thickness of the layer should satisfy the following condition

$$d = m\lambda / 2n_2$$

From this analysis we can also conclude that for a given thickness and refractive index of a dielectric layer, there will be maximum transmission at the following wavelengths:

$$\lambda_{\text{peak}} = 2n_2 d / m$$

In the case, where the forward moving waves add up out of phase the optical path length differences equal an uneven number of half-wavelengths:

$$\Delta\text{OPL} = (2m-1)\lambda/2$$

as illustrated on Figure 7.

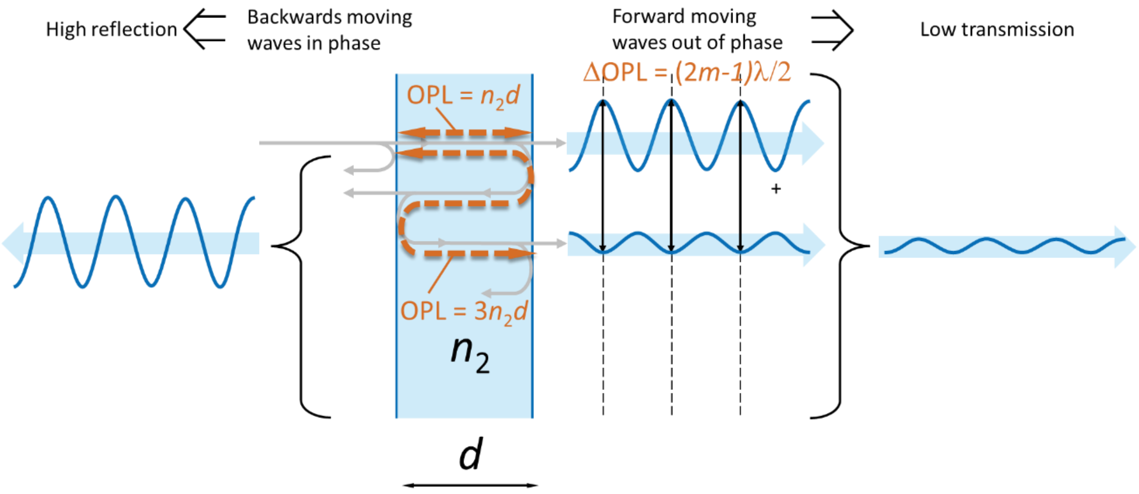

Figure 7: OPLs for the first two passes through the dielectric layer. In case the waves add up out of phase ΔOPL = (2m-1)λ/2.

The actual maximum and minimum transmitted light depends on the reflection coefficient R_i at each of the two surfaces i = I and II, respectively. For the case where $n_1 = n_3$, the two reflection coefficients become identical, hence $R_I = R_{II} = R$. In this simplified case, it can be shown that the total transmission T_{tot} is given by the following equation

$$T_{tot} = \frac{(1-R)^2}{1+R^2 \pm 2R\cos\left(2\frac{2\pi}{\lambda}n_2 d\right)}$$

where the plus-sign in the denominator is used when $n_2 < n_1$ and the minus sign when $n_2 > n_1$.

Multi-layer coatings

As just described a single dielectric coating layer will produce an optical filter with maximum transmission at the wavelengths given by $\lambda_{peak} = 2n_2d / m$, where m is an integer. The spectral transmission of such a filter is shown on Figure 8 for the case where d = 1 micrometer and n_2 = 1.5 and the reflection R at each surface is 4 %. Although the filter does provide maximum and minimum transmissions as expected it is not a very useful filter.

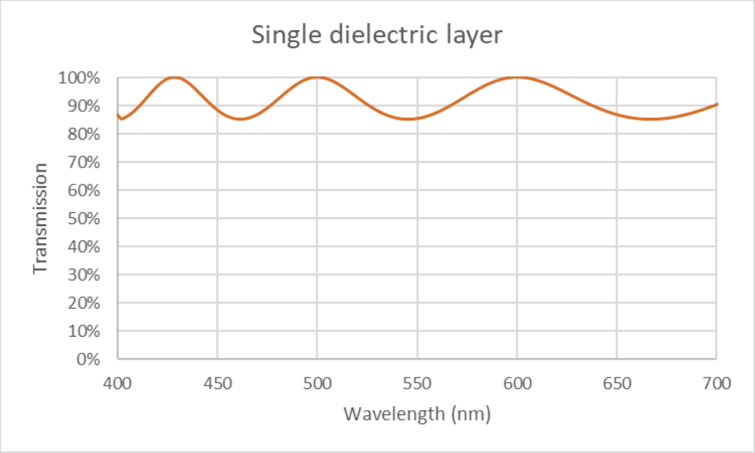

Figure 8: Transmission through single layer dielectric filter.

A common type of filter function is Anti-Reflection (AR) coatings, which reduce the reflection at an air-glass interface and thereby improves the transmission. A simple AR-coating can be made with a single layer of MgF_2, which has a refractive index around 1.38, and a thickness of $\lambda/4$. Such a MgF_2 layer can reduce the reflection from 4% to around 1%.

To design a more advanced filter, it is necessary to add many dielectric layers of alternating high and low refractive index. In this way it is possible to make a bandpass filter like shown on Figure 9 with a nice high transmission in the pass-band and strong rejection outside the passband.

Other common types of optical filter functions are long wave pass and short wave pass edge filters, notch filters, and intensity, wavelength or polarization beam splitters. Also, even more advanced filters can be manufacutred by combining several filter functions in the same coating. One example is multi-bandpass filters which provides several wavelengths pass bands in the same filter.

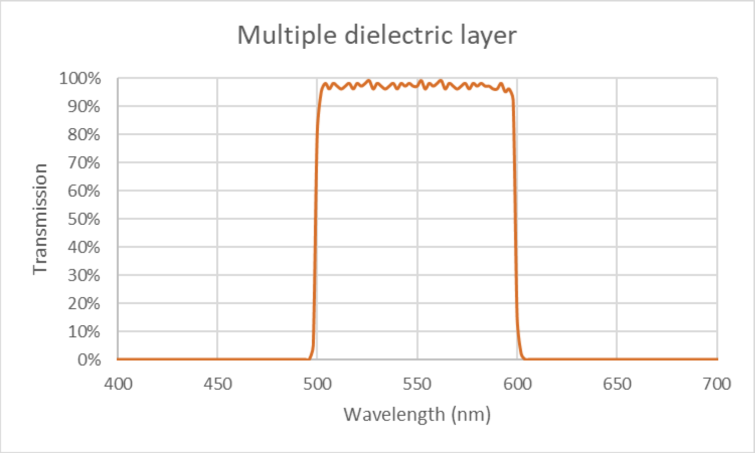

Figure 9: Transmission through multi-layer dielectric filter.