The gradient index filter: An overview

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Abstract

The unique properties of the gradient index filters are demonstrated by a combination of practical examples and theory. The paper contains guidance for the future user of the new type of optical filters, and describes the principal aspects of the synthesis technique used for the design of the coatings. The contributions made by DELTA Light & Optics to the development of the technique are outlined. Finally the text deals with the realisation of the complex thin-film structures.

Introduction

Most optical systems contain components coated with dielectric coatings. The best known type of coating is the antireflection coating that decreases the reflection from an optical surface and protects it against scratches. Other well-known types of coatings are bandpass filters, edge filters, laser mirrors and beamsplitters.

In some applications the coatings are inexpensive and simple parts. However, the thin film devices may also be some of the most important and advanced parts in a system. The new gradient index filters are typically crucial parts of high tech equipment. The easiest way to give an impression of the properties of the new filters is to describe some of the filters developed.

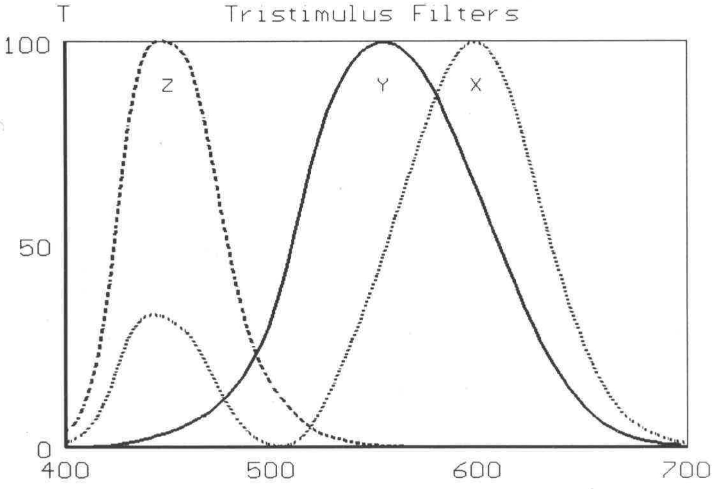

Figure 1. Desired response curves for a system measuring the tristimulus values X,Y and Z that express the colour of a test sample in the CIE colour system.

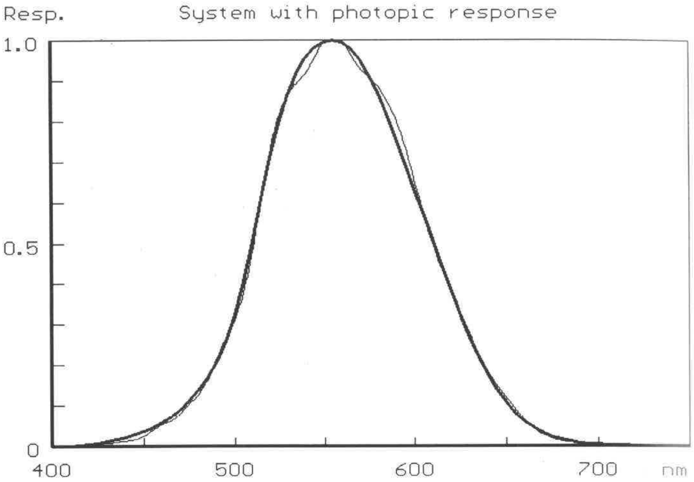

Figure 2. Obtained correlation between the V-lambda curve (thick solid curve) expressing the colour sensitivity of the human eye, and the response of a sensor system with a photopic response (thin solid curve). The F1-error of the system is 3 percent.

The main product is a set of filters developed for a colour analyser from Philips TV Test Equipment in Denmark. Three different filters were developed that make the spectral characteristic of silicon photo-diodes and optical parts belonging to them match the tristimulus curves (X, Y, Z) shown in Figure 1. These response curves are needed to determine the colour coordinates of light in the CIE colour system[1]. The compact colour analyser is used to calibrate the colours of monitors and television sets in production lines, at TV-stations and in the repro industry.

We have also developed photopic filters for various sensor systems used for the measurement of the brightness of light. One photopic filter was developed for our new Retrosign instrument that measures the retro-reflection from road signs. In this instrument the light is sensed by a silicon photodiode. Another one is developed for the LTL2000 instrument used for measurement of the retro-reflection from road markings. In this case the detector is a PMT. A special version is developed for a mega-

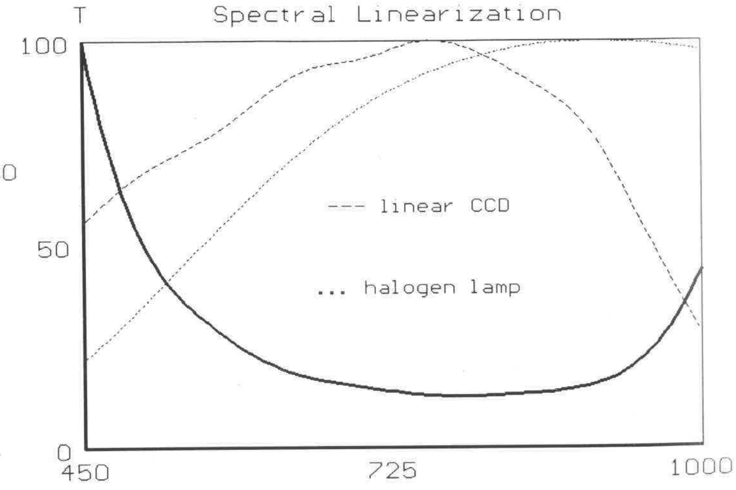

Figure 3. The dynamic range of a spectrometer system containing a halogen lamp and a linear CCD array is enhanced by a spectral linearization.

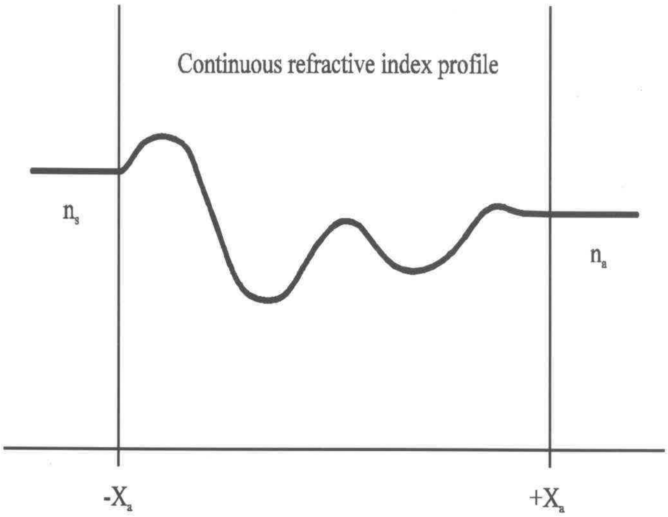

Figure 4. Model of an inhomogeneous optical coating. The continuous refractive index profile is adapted to homogeneous boundary media.

pixel CCD-camera that will be able to make spot measurements in the viewing field. Figure 2 shows the correlation that has been obtained between the colour sensitivity of the human eye (thick solid curve) and the response of the sensing system (thin solid curve). The so-called F1-error[2] of the system is approximately 3%. The QD30 instrument used to measure the reflection properties of roads and road markings as seen by the driver in daylight or diffused light is equipped with a gradient index filter that corrects the emission of a halogen lamp to match the spectrum of the D65 source as well as the sensitivity of a silicon photodiode to match the colour sensitivity of the human eye.

The first filter developed was used for linearization of the spectral sensitivity of a fibre optic spectrophotometer containing a halogen lamp and a linear CCD-array[3] (see Figure 3). This filter increased the dynamic range of the spectrometer system by nearly a decade.

An important group of the gradient index filters are the so-called rugate filters that are single or multiple notch filters without higher order reflections. These filters are mainly used for eye and sensor protection in military equipment. The rugate filters are not part of our program today. However, theoretical contributions have been made to this field of endeavor[3,4,5]

The examples listed give you an impression of the spectral characteristics of the gradient index filters and their application. The filters track a desired path as precisely as possible in a broad spectral range typically covering more than the whole visible range. It is important to note that the filters typically are custom-designed for a specific instrument.

The transmission curve of a gradient index filter typically differs from those of the classical filters that are characterized by steep transitions between the transmitting and rejecting spectral ranges and less strict tolerances on the transmission levels. The characteristics are obtained at the cost of an increased complexity in the layered structures and smaller tolerances on the process parameters. Furthermore special design techniques are required.

In the following, a brief description is given of the design and production techniques needed to produce filters with the desired characteristics.

How to determine which coating is needed!

The customer for a gradient index filter needs a specific spectral response or emission from an optical system. This means that we must determine the coating needed for the system to obtain this performance. In 1995, software was developed that can be used to calculate the required transmission of the missing coating, when the rest of the components have been characterized. It is possible to work with multiple reflections between parts, and with non-collimated and polarized light. At the same time DELTA Light & Optics is ready to assist people with measurements of the spectral response of sensors, the transmission through materials and components and the spectral emission of light sources.

Synthesis of the desired coating

The synthesis of advanced optical coatings is as complex as lens design. The typical coating is a multi-layered stack of homogeneous layers of alternating high and low refractive index[6]. Due to the refractive index contrast, the light is reflected in the layers that typically are thinner than the wavelength of light. The transmission changes due to interference between the reflected light waves. An all-dielectric coating absorbs in principle no light - it is either reflected or transmitted. The coating is sensitive to the angle of incidence of the light and to the state of polarisation[6]. Because gradient index filters typically cover a broad spectral range, it is normally necessary to limit the angle of incidence of light to be within +/- 20 degrees.

The classical way to design an optical multi-layer is to make a clever choice of a starting design and to optimize the thickness of the individual layers[7,8,9]. Edge-filters, notch-filters, mirrors, colour beamsplitters and bandpass filters are examples of widely used coatings that can be optimized from quarterwave-related stacks. The elimination of ripples in the passbands is mainly a question of obtaining a good adaptation between the multi-layer and the surrounding media - and depends mostly on the thickness of the outermost layers in the stack. The presence of series of layers of equal optical thickness in the classical filters make them less sensitive to minor errors in the produced layer-thicknesses than filters composed of layers of different thickness[10].

It would not be realistic to try to design a coating with a spectral performance, which differs considerably from that of a laser mirror or a bandpasfilter with the technique just described. The desired transmission curve of a typical gradient index filter affords a much larger number of layers that all have a different thickness. Some will be in the range of a quarter of a wavelength whereas others will be 20 times thinner or less. It is totally unrealistic to make a qualified guess of the layer-code of such a coating. A different synthesis technique is needed for this purpose. Since 1990, DELTA Light & Optics has contributed to the development of a synthesis technique called the inverse Fourier transform technique. In the following a brief description of the technique is presented. A more comprehensive presentation is outside the scope of this text.

Fundamentals of optical thin film coatings

We will examine the reflection and transmission of light incident normally to an inhomogeneous dielectric thin-film. The refractive index $n(x)$ inside the thin-film is a function of the coordinate x that is $-x_a$ at the substrate and $+x_a$ at the boundary to the medium of incidence (see Figure 4). It is assumed that the refractive index of the layer passes continuously into the refractive indices of the boundary media

$$\left.\begin{aligned} n(-x_a) &= n_s \\ n(+x_a) &= n_a \end{aligned}\right\} \tag{1}$$

It is convenient to consider the refractive index profile as a function of twice the optical thickness[4,5,11,12].

$$x = 2\int_0^z n(u)\cos\theta_p \, du \tag{2}$$

where θ_p is the angle of propagation in the medium of index $n(x)$, and where z is the geometrical coordinate within the layer. Assuming that an electromagnetic wave with a time dependence $\exp(i\omega t)$ is incident from the right, Maxwell's equations can be reduced to[8,13]

$$\left.\begin{aligned} \frac{du}{dx} &= ik\, n^{-1}(x)\, v \\ \frac{dv}{dx} &= ik\, n(x)\, u \end{aligned}\right\} \tag{3}$$

where $k = 2\pi / \lambda$ is the wavenumber, and $u=u(z,k)$ and $v=v(z,k)$ are the complex electric and magnetic amplitudes of the waves. The amplitude reflectance and transmittance of the optical coating can be expressed through the solution of Eqs. (3) with the boundary conditions

$$u(-x_a,k) = 1 \quad \text{and} \quad v(-x_a,k) = n_s \tag{4}$$

The expressions have the following form[13].

$$\left.\begin{aligned} r(k) &= \frac{n_a\, u(x_a,k) - v(x_a,k)}{n_a\, u(x_a,k) + v(x_a,k)} \\ t(k) &= \frac{2n_a}{n_a\, u(x_a,k) + v(x_a,k)} \end{aligned}\right\} \tag{5}$$

The corresponding expressions for the intensity reflectance and transmittance are as follows

$$\left.\begin{aligned} R(k) &= |r(k)|^2 \\ T(k) &= \frac{n_s}{n_a}|t(k)|^2 \end{aligned}\right\} \tag{6}$$

In the special case where the layer is homogeneous, u and v can be expressed by

$$\begin{pmatrix} u \\ v \end{pmatrix}_{x=x_j} = \mathbf{M}_j \begin{pmatrix} u \\ v \end{pmatrix}_{x=-x_j} \tag{7}$$

where

$$\mathbf{M}_j = \begin{pmatrix} \cos\varphi_j & \dfrac{i}{n_j}\sin\varphi_j \\ i\, n_j \sin\varphi_j & \cos\varphi_j \end{pmatrix} \tag{8}$$

is called the characteristic matrix of the layer indexed j. $\varphi_j = x_j/2$ is the phase thickness of the layer indexed j.

The characteristic matrices are multiplied when a coating consists of multiple homogeneous sub-layers.

$$\begin{pmatrix} u \\ v \end{pmatrix}_{x=x_a} = \mathbf{M}_m \mathbf{M}_{m-1} \cdots\cdots \mathbf{M}_1 \begin{pmatrix} u \\ v \end{pmatrix}_{x=-x_a} \tag{9}$$

where the layers are indexed from the substrate.[8] Please note that the indexing varies in literature![6,8] In this text the layers are indexed as deposited.

It is common to use Eqs. (5 - 9) to calculate the characteristics of both multilayers[6] and inhomogeneous coatings[3,14] when the refractive index profile is known. This is called to perform an analysis. In case of an inhomogeneous coating, the layer is treated as consisting of a large number of thin homogeneous layers.

The inverse Fourier transform technique

The inverse problem is to synthesize a coating with a desired optical performance. In this case we have to find a refractive index profile which gives us the desired performance.

The function

$$f(k) = \left(\frac{n_a}{n_s}\right)^{1/2} \frac{r(k)}{t(k)} \tag{10}$$

is one of the elements in the so-called transfer matrix[8] that relates the amplitudes of the waves travelling in the negative and the positive directions in the medium of incidence to the corresponding amplitudes in the substrate. It belongs to a special class of functions whose Fourier transforms are different from zero only over a limited range. In the case of a <u>continuous</u> refractive index profile the function can be represented in the following way[11,13]

$$f(k) = \int_{-x_a}^{x_a} F(x)\exp(ikx)\, dx \tag{11}$$

The modulus of the $f(k)$ function is usually called the Q-function

$$Q(k) = |f(k)| = \left(\frac{R(k)}{T(k)}\right)^{1/2} \tag{12}$$

The Fourier transform synthesis method is based on an approximate representation of the exact Fourier transform (11). In the case of a smooth refractive index profile (continuous curve) and a not too high reflectance, the following approximate formula turns out to work well[4,11,12,13,15,16]

$$f(k)=Q(k)\exp(i\Phi(k))=\frac{1}{2}\int_{-\infty}^{\infty}\frac{n'(x)}{n(x)}\exp(ikx)\,dx \tag{13}$$

$\Phi(k)$ is the phase function $\arg[f(k)]$ and $n'(x)$ symbolizes the derivative of $n(x)$, $dn(x)/dx$.

The expansion of the integration limits from $-x_a$, $+x_a$ to $-\infty$ to $+\infty$ is possible because $n'(x)$ is zero outside the inhomogeneous layer. The substitution of the inverse Fourier transform of $f(k)$ by one half of the logarithmic derivative of the refractive index is an approximation. Inverting the Fourier integral (13), we get

$$\frac{dn(x)}{dx}\frac{1}{2n(x)}=\frac{1}{2\pi}\int_{-\infty}^{\infty}Q(k)\,\exp[i(\Phi(k)-kx)]\,dk \tag{14}$$

Since the transmission function of a coating is always an even function of the frequency, $\Phi(k)$ must be an odd function $\Phi(-k) = -\Phi(k)$. By integrating both sides of Eqs. (14) we get

$$n(x)=\exp\left[\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{Q(k)}{k}\,i\exp[i(\Phi(k)-kx)]\,dk\right] \tag{15}$$

that can be simplified to become[5]

$$n(x)=\exp\left[\frac{2}{\pi}\int_{0}^{\infty}\frac{Q(k)}{k}\,\sin[(\Phi(k)-kx)]\,dk\right] \tag{16}$$

as cosine is an even function and sine an uneven function.

Eqs. (16) is the central equation in the so-called Inverse Fourier Transform Synthesis method[4,11,12]. The refractive index profile is always centred onto the limited refractive index range of the applied thin film materials prior to the calculation of the corresponding transmission curve. The centring performed by multiplying with a factor G does not affect the obtained transmission curve[4,11]

$$G=\left(\frac{n_L n_H}{n(x)_{min}\,n(x)_{max}}\right)^{1/2} \tag{17}$$

Here $n(x)_{min}$ and $n(x)_{max}$ are the extremal values of $n(x)$ calculated from Eqs. (16).

The Q-function and the phase-function

The performance of the inhomogeneous coatings designed with Eqs. (16) is good as long as the reflection is low. Figure 5 shows an example of a desired transmission curve of a GIF and the corresponding refractive index profile and transmission curve. Dobrowolski et al.[12,16,17] have demonstrated this several times during the years. However, severe deviations arise when trying to design coatings with high reflection in different ranges[4,15,16]. Furthermore, the refractive index modulations tend to become unrealistically large near the centre of the coating.

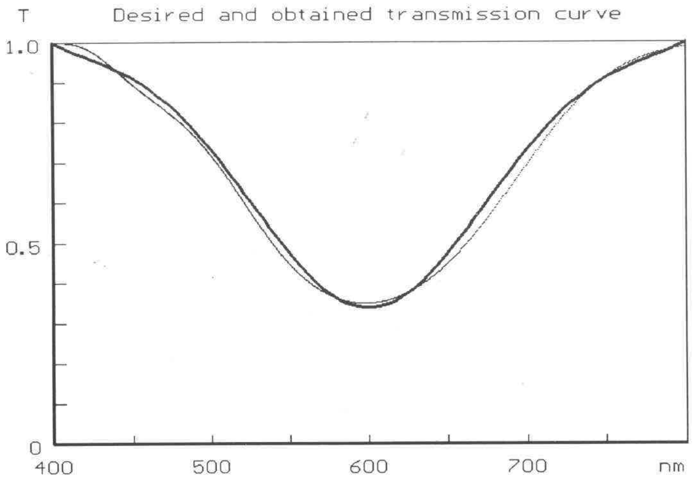

Figure 5a. The performance of the inhomogeneous coating obtained with Eqs.(12) and (16) is acceptable as long as the reflection is low.

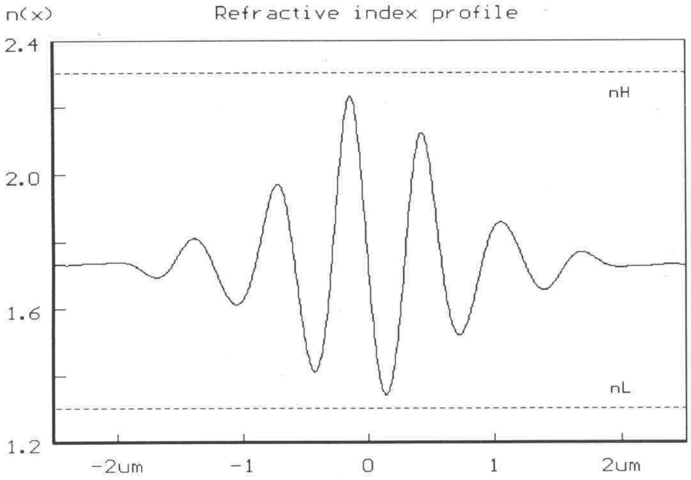

Figure 5b. Refractive index profile corresponding to the inhomogeneous coating. Dashed lines indicate the refractive indices of the high index material ZnS and the low index material Misch Fluoride.

The refractive index modulations can normally by reduced by a proper choice of the phase function. The most efficient expression so far is[3]

$$\Phi(k)=\frac{\pi k}{k_{max}+k_{min}}-\frac{\pi}{2}\sin\left[N\pi\frac{k-k_{min}}{k_{max}-k_{min}}\right] \tag{18}$$

Where N is a real number that is typically in the 1 - 5 range; k_{min} and k_{max} are the spectrally limiting wavenumbers. Lately some prefer to include the phase condition directly into the Q-function and treat this as a complex quantity[18].

The accuracy of Eqs. (16) depends strongly on the choice of the Q-function. Eqs. (12) is not a good choice at high reflection. The discussion of the optimal choice of Q-function has been an important issue within the last decade[13,14,15,16,18,19]. One approach has been to work with coatings that can be analysed with

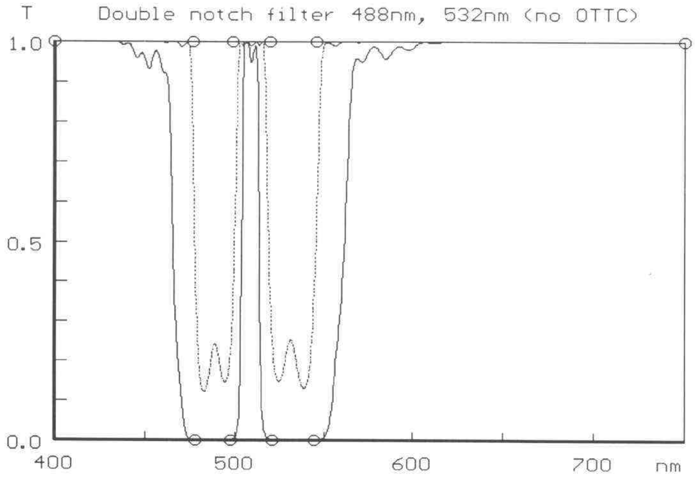

Figure 6. Omitting the OTTC corrections the calculations get unstable when the desired transmission curve contains steep transitions between high and low reflection. Circles indicate the values of the desired double notch filter.
The dotted curve shows the initial result obtained with Eqs. (16) and (20) whereas the solid curve shows the result after 10 iterations.

different methods[14,19]. Bovard claimed the following expression to be less approximate in the case of quarterwave optical coatings[14].

$$Q_2(k) = \tanh^{-1}\left[\sqrt{R(k)}\right] \tag{19}$$

However, despite Bovard's attempts to find more accurate expressions, he concluded October -93 that the Q-functions remain approximate[20]. In the meantime, the author proposed on Sept -92[4] and August -93[19] that it seems to be a more fruitful idea to search for Q-functions that are as well suited for numerical iterations[19] as possible, and it was shown that the following Q-function has better properties in this respect[19]

$$Q_3(k) = \frac{Q_2(k)}{\frac{1}{\pi}Q_2(k) + \sqrt{1 + \left(\frac{1}{\pi}Q_2(k)\right)^2}} \tag{20}$$

The iteration technique was already mentioned briefly by Sossi in his original work.[11] However, the technique was further developed by us, and it was shown that it is possible to compensate for a number of errors inherent in the inverse Fouriers transform technique by changing the Q-function iteratively[3,4,19]. The Q_3-function performs well in a closed loop optimization process (iterations with unchanged conditions) and this type of optimization is very fast compared to merit function based optimization techniques.[8] Further results with iterations of the Q-function will be published soon by P. G. Verly[21]

Truncation of the optical thickness

The inversion of the Fourier integral in Eqs. (14) made the refractive index modulation become non-zero at all thicknesses of the thin-film. However, in practice the designer would seek to minimize the optical thickness of the coating for economical reasons as a prolonged process time is equal to expenses. Besides, the obtainable layer-thicknesses are limited by material and process dependent parameters like internal stress and crucible capacity. Truncating the optical thickness, errors are introduced in the obtained transmission curve. In some cases it is possible to remove the deviations by performing the proposed iterations on the Q-function. However, the calculations get unstable if the desired transmission curve contains steep transitions between high and low reflection[4]. This is illustrated in Figure 6 where the dotted curve is the result obtained initially and where the solid curve is the result after 10 iterations. The circles indicate the values of the desired transmission curve. Until recently, this made the inverse Fourier transform technique

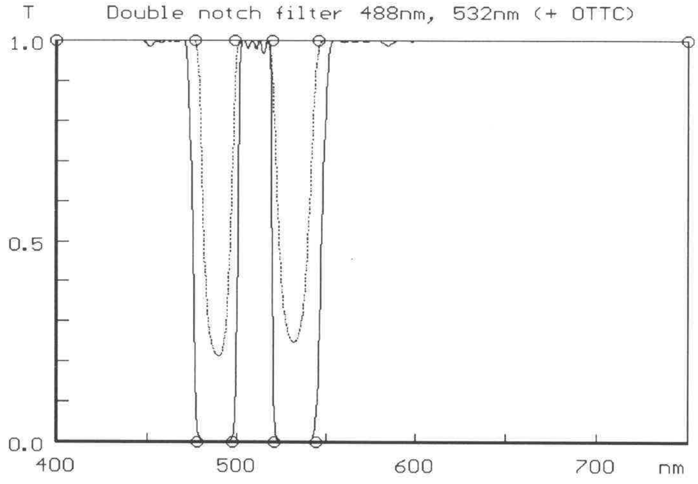

Figure 7a. Applying the OTTC corrections the calculations get stable. The circles indicate the values of the desired double notch filter. The dotted curve shows the initial result obtained with Eqs. (16) and (20) whereas the solid curve shows the result after 22 iterations.

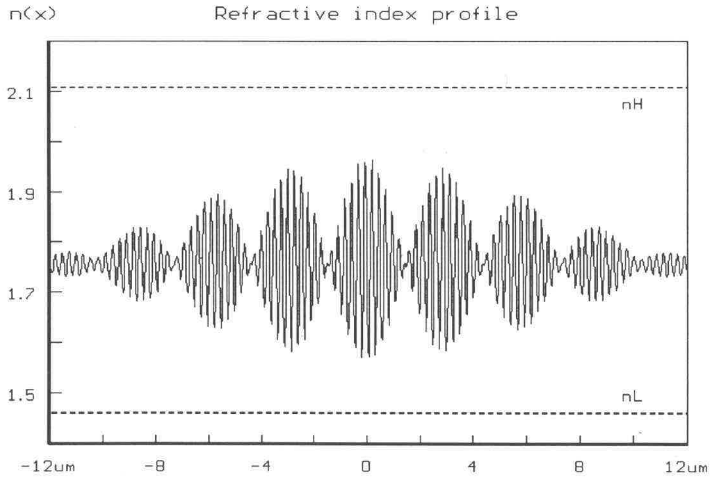

Figure 7b. Refractive index profile corresponding to the solid curve in figure 7a. The dashed lines indicate the refractive indices of the high index material Substance1 and the low index material SiO_2. The optical thickness is 24 μm.

useless for the the design of important devices like head-up displays and eye protecting coatings for military purposes. Researchers like W. Southwell[22,24], W.E.Johnson[23], R.L. Hall[24] and B. Bovard[20] have been working with a different forward designing technique for this purpose. Single and multiple rejection filters are constructed by adding sine varying refractive index profiles (rugates)

$$n_i = n_a + \frac{1}{2} n_p \sin(k_i - \Phi_i) \tag{21}$$

A number of interesting papers were published on this issue. The author utilized some of the results to develop a correction technique for the inverse Fouriers transform technique which helps stabilize the iterations on truncated profiles[4]. The Optical Thickness Truncation Compensating (OTTC) technique means that it is now possible to design highly reflecting filters with the inverse Fourier transform technique.[4,19] The solid curve in Figure 7 was obtained after 22 iterations. The corresponding refractive index profile is also shown. The slashed lines indicate the refractive indices of the high index material Substance1 and the low index material SiO_2.

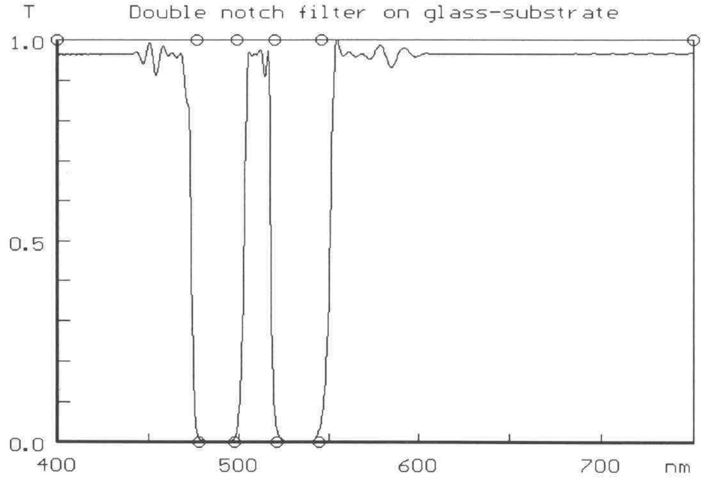

Figure 8a. Choosing boundary media like glass and air introduces ripples caused by misadaptation. However, the ripples are nearly removed again when overlaying the quintic matching layers and performing further iterations. In this case the total number of iterations was 50.

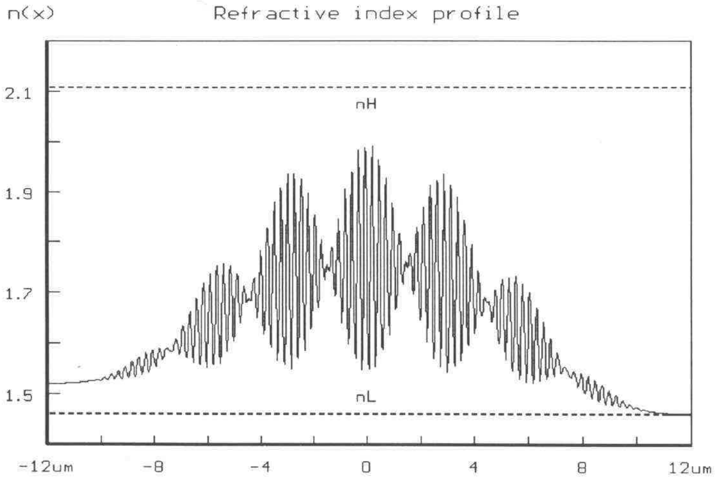

Figure 8b. Refractive index profile corresponding to figure 8a. The quintic matching layers were overlayed from the boundaries to the center of the coating. The carrying substrate is made of glass and the external medium is air.

Taking dispersion into account

It was assumed that the refractive indices of the thin-film materials are constant when Eqs. (16) was derived. However, real thin film materials are dispersive and it is necessary to take this into account to design GIFs with a desired broadband spectral performance. A technique was developed that made it possible to introduce a synthetic dispersion in the calculations when the thin film materials are known[3].

Introducing real substrates

In practice a thin film must be deposited on a carrying material. The substrates normally introduce ripples on the transmission curve as a consequence of misadaptation between the thin film and the surroundings. Sometimes it is possible to suppress the ripples by performing iterations on the Q-function. Another possibility is to overlay quintic matching layers at the ends of the GIF[4,24]. Figure 8 shows the transmission curve obtained in this case and the corresponding refractive index profile.

Inhomogeneous and quasi-inhomogeneous optical coatings

The advantages of a completely inhomogeneous optical coating compared to a multi-layer one are mainly

- that it is the only possible way to design a truly single notch filter.
- that the maximum strength of the electric field is lower because there are no discontinuities in the refractive index.

In practice this is of little importance, and the rugate filters and gradient index filters available today are digital counterparts to the continuous profiles or so-called quasi-inhomogeneous coatings[25]. The conversion technique we apply[3,19] is based on the application of double layer equivalents. It implies that the continuous refractive index profile is divided into *N* relatively thin layer elements prior to the conversion[3]

$$OT = OT_{tot} / N = C_p n_p t_p << \lambda \tag{22}$$

where

OT	is the optical thickness of the sublayer
OT_{tot}	is the total optical thickness of the inhomogeneous layer
N	is the number of sampling elements
p	is the indexing number of the layer
n_p	is the refractive index of the sublayer indexed p
t_p	is the physical thickness of the sublayer indexed p
λ	is the wavelength of the incident light

and where C_p is the cosine to the angle of refraction in the sublayer as expressed by

$$C_p = \cos\theta_p \tag{23}$$

The angle of propagation is given by Snell's law

$$n_p \sin\theta_p = n_0 \sin\theta_0 , \tag{24}$$

where the label 0 denotes the medium of incidence. The criterion for the conversion as expressed by Eqs. (22) is a sampling criterion that originates from first order approximations performed onto sine and cosine functions in Refs. 3 and 19.

$$\mathbf{M}_p = \begin{pmatrix} \cos\varphi_p & \frac{i}{\eta_p}\sin\varphi_p \\ i\eta_p \sin\varphi_p & \cos\varphi_p \end{pmatrix} \approx \begin{pmatrix} 1 & i\frac{\varphi_p}{\eta_p} \\ i\eta_p\varphi_p & 1 \end{pmatrix} \tag{25}$$

where η_p symbolizes the optical admittance of the layer indexed p:

$$\eta_p = n_p X_p \tag{26}$$

where

$$\left.\begin{aligned} X_p &= C_p && \text{for S-polarization} \\ X_p &= 1/C_p && \text{for P-polarization} \end{aligned}\right\} \tag{27}$$

The conversion is performed by converting each sublayer, indexed p, into a combination of a thin layer of high refractive index n_H and physical thickness t_{PH} and a thin layer of low refractive index n_L and physical thickness t_{PL}

$$\left.\begin{aligned} t_{PH} &= \frac{X_H}{C_H}\frac{\eta_p^2-\eta_L^2}{\eta_H^2-\eta_L^2}\frac{OT}{\eta_p} \\ t_{PL} &= \frac{X_L}{C_L}\frac{\eta_H^2-\eta_p^2}{\eta_H^2-\eta_L^2}\frac{OT}{\eta_p} \end{aligned}\right\} \tag{28}$$

The developed conversion technique is approximate, and the deviations increase with the relative optical thickness of the unconverted layers[3,19]. However, it can be shown that it is possible to convert unexpected thick layer elements by converting all uneven indexed sublayers into HL equivalents and all even indexed sublayers into LH equivalents[19]. The number of layers in the two index solution at the same time decreases to $N + 1$ which should be compared to the total number of layers in a conventional Herpin conversion which is somewhere between $(2N+1)$ and $3N$.[19] The transformation operation is single stepped and suited for fast closed loop optimization of the approximate two indexed solution (see Figure 9). The optimum number of layers in the two-index solution is found to be approximately four times the number of cycles in the profile. A considerably larger number of layers is not found to be of any advantage. At too low numbers strong reflections occur at the low wavelength range.[19]

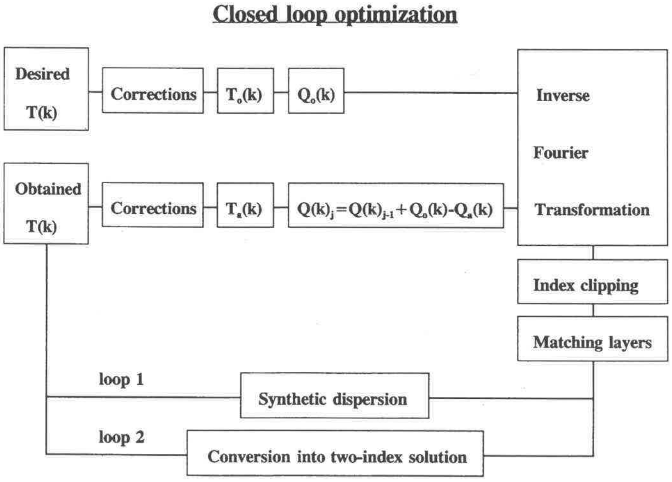

Figure 9. Diagram illustrating the closed loop optimization technique.

Merit function based optimization

It is possible to optimize the two-indexed coating further (than can be obtained with the closed loop optimization) by performing merit function based optimization.[6,7,8,9] In this case, it is investigated whether it is of advantage to make changes in the layer thicknesses by performing differentiations on the system matrix (see Eqs. 9). The large number of layers in the coating makes this type of optimization very time consuming. However, lately second order optimization techniques were implemented[9] that speed up the convergence of the calculations. The development of up-to-date design software is an ongoing process of increasing importance. Today we have at our disposal advanced software for the optimization and synthesis of complex coatings

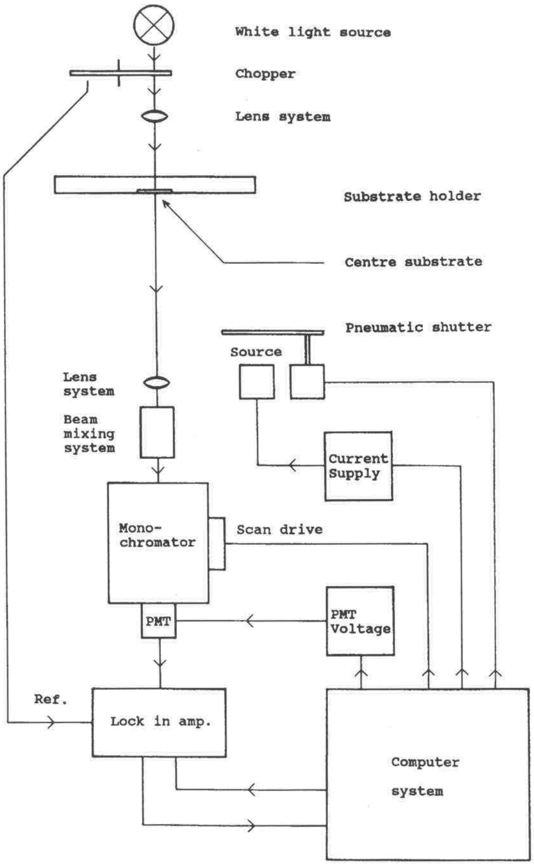

Figure 10. Schematic illustration of the developed computer controlled system. The computer collects data from the optical measuring system and controls the coating process on the basis of these data.

as well as of whole filter systems. Efficient second order optimization techniques utilizing large two-dimensional Hesse matrices and synthesis techniques like the inverse Fourier transform technique[5,13,19] and the needle optimization technique[8,13] are applied daily. The possibility of accessing the code whenever it is needed and to make changes and develop it is crucial in an environment where research is performed daily and advanced coatings are developed for industrial partners all over the world.

Deposition of quasi inhomogeneous optical coatings

The deposition of the quasi-inhomogeneous optical coatings is a difficult task that only very few manage. It still seems as if DELTA Light & Optics is the only company that makes this type of coating for nonmilitary applications.

The quasi-inhomogeneous coatings from DELTA Light & Optics that have got the product name “Gradient Index Filters” are soft-coated from the high indexed material Zinc Sulfide and a low-indexed fluorid-material. We use a Balzers BAK600 coating machine for this purpose. The coating is deposited on glass-substrates placed in a rotating substrate holder. The on-line measurement of the optical thickness of the different layers is based on an optical transmission measurement of mono-chromatic light through the centre substrate (see Figure 10). The transmission through the filter changes during the deposition of each layer due to the interference in the deposited multi-layer. The deposition is controlled by comparing the changes in the transmission with computer predicted data. This means that the measurement of the thickness of the layers is indirect and that the obtainable precision depends on how well you manage to simulate the optical performance of the layers in the structure as well as on the precision of the measurements. If a layer has got an optical thickness or a refractive index differing from what was predicted this will influence the rest of the coating process. Turning points on the transmission curve from the fixed wavelength measurement gives you the possibility to make some on-line corrections. Some of the easy aspects about the classical edge and bandpass-filters are that the layer thicknesses are large enough that you will have turning points in almost all layers and that they are relatively insensitive to errors. It is usually possible to deposit the coating without changing the steering wavelength more than a few times. This is absolutely not the case when working with the gradient index filters.

Most of the layers in a quasi inhomogeneous optical coating are thinner than a tenth of a wavelength - and some of them six times less than that. At the same time the optical thickness of the layers must be correct within a few percent. It is necessary to change the steering wavelength frequently to obtain this precision - 40 to 50 shifts during the deposition is common and the wavelength range is typically 200 nms wide. The deposition process typically takes more than seven hours. All the same it is only possible to obtain turning points and the possibility to do on-line corrections in some of the layers. Therefore it has been necessary to develop advanced mathematical tools to generate optimal steering data for the process-system. Today this part of the design process is far the most complicated and time-consuming and we try to develop the theoretical tools further to lower the costs of the designing process.

Normally, the deposition of filters is controlled by an operator. However, this turned out to be unrealistic when producing the gradient index filters. At the end of 1993, we succeeded in developing a computer-based system to control and steer the deposition of the gradient index filters on the basis of the data from the transmission measuring system. Figure 10 is a schematic illustration of this system. The developed system is also used for

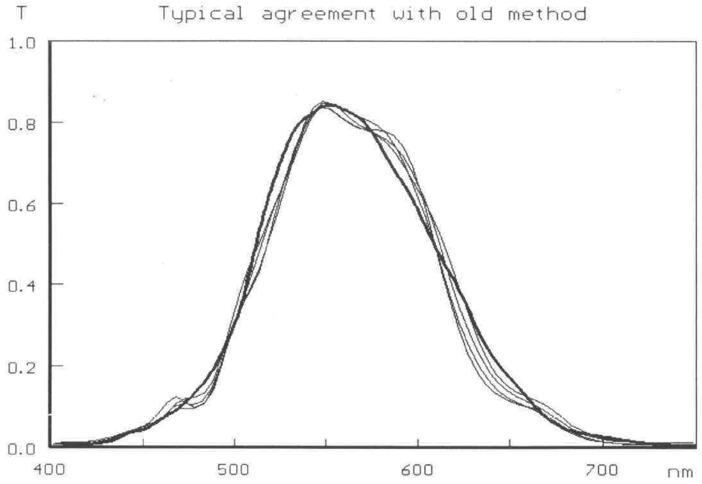

Figure 11: Thick solid curve showing the desired transmission curve of a smart-filter that gives a detector system a V-lambda response. The thin solid curves show some obtained transmission curves. The systematic deviations are due to differences in the packing densities of the layers not taken into account in the designing process.

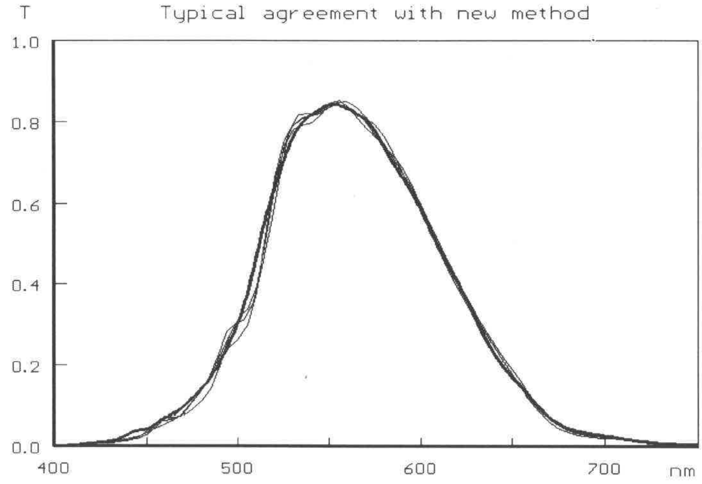

Figure 12: Thick solid curve shows the desired transmission curve of a smart-filter that gives a silicon photodiode a V-lambda response when used with a 1 mm thick BG40 coloured glass. The thin solid curves show some of the obtained transmission curves for the new design where the differences in the packing densities of the different layers is taken into account. The F1-error of the system that is simulating the spectral sensitivity of the human eye is 3.3% ± 0.5%.

the production of a series of more traditional soft-coated filters. In contrast to an operator who operates the system manually, the computer system only makes objectively based on-line corrections. At the same time, the system is capable of extracting the speed of deposition from the optical measurement and servo-controlling the speed of deposition. An immediate effect was an improved uniformity of the filters from different productions. In this way it was made visible that the deviations between the predicted and the actual variations of the measuring signals were very systematic. Concerning the gradient index filters, the steering problems resulted in distinct and systematic discrepancies in spectral performance of the finished filters (see Figure 11). Calculations on the computer made us suspect that the problem was variations in the packing densities/refractive indices of the layers.[2] The repeatability of the deviations during the depositions seemed, however, to indicate that it would be possible to take the variations into account if the relation between the process-parameters and the variations in the packing density of the thin films could be understood. In the period 1994 - 95, a number of experiments were made to search for such relations.[2] We found that the packing density of the ZnS-layers did depend on the thickness of the underlying thin-film structure, whereas the packing density of the fluoride-layers was determined by the packing density of the underlying ZnS-layer and the substrate temperature at the beginning of the layer[2]. The repeatability of the coating process that is obtainable with the computer controlled system made it possible mathematically to predict the progress of the substrate temperature with a tolerance of about 5 degrees on the basis of the layer-code and the desired speed of deposition.[2] This means that it is now possible to predict the packing density and the refractive index of the layers in the coating in the design stage. Figure 12 gives an impression of the correlation that was obtained as a result of this work that also lead to an improved quality of our traditional soft-coated coatings.

Future works

Another possibility to overcome the problems with non-constant packing densities, would of course be to use a coating technique that stabilizes it. Possible techniques would be sputtering, ion assisted deposition and chemical vapour deposition. Sputtering and CVD processes are not really suited for the deposition of larger batches, whereas ion assisted deposition has attracted much attention recently.[26,27] DELTA Light & Optics did install a Mark II ion gun for ion assisted deposition of hard oxide materials in 1995, and we are making good progress with the equipment. We hope to transfer the technique to the production of the gradient index filters within the next years.

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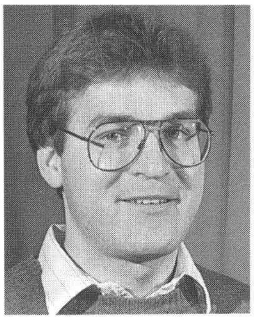

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