Gradient-index filters: designing filters with steep skirts, high reflection, and quintic matching layers

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Gradient-index filters from Lys & Optik were developed on the basis of an inverse Fourier transformation of the desired spectral characteristics. Filter designs with steep skirts and high reflection are possible because of a new correction method where analytical calculations for rugate structures are used as a basis for the initial corrections of the input characteristics. Overlaying of the quintic matching layers and successive approximation methods are used for the suppression of ripples on the transmission curve resulting from misadaptation between the gradient-index thin film and the surrounding media.

Key words: Gradient-index filters, inverse Fourier transformation, rugate, quintic matching layers, correction.

Introduction

Sossi and Kard[1-3] have shown that it is possible to relate the spectral transmittance of an inhomogeneous nonabsorbing layer to its refractive-index profile $n(x)$ by the approximate expression

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}n}{\mathrm{d}x} \frac{1}{2n} \exp(ikx)\mathrm{d}x = Q(k)\exp[i\Phi(k)], \quad (1)$$

where k is the wave number.

$$k = 2\pi/\lambda, \quad (2)$$

where λ is the wavelength of light in vacuum.

The x is twice the optical distance from the geometrical center of the inhomogeneous layer. Thus

$$x = 2\int_0^z n(u)\mathrm{d}u, \quad (3)$$

where z is the geometrical coordinate within the layer.

$Q(k)$ is a suitably even function of the desired transmittance $T(k)$. Sossi introduced the following expression, which was also used by Dobrowolski and Lowe[4]:

$$Q(k) = \{1/2[1/T(k) - T(k)]\}^{1/2}. \quad (4)$$

However, this expression does not work when $T(k)$ is close to zero. In this case it is much better to use the following expression, which was derived by Bovard[5]:

$$Q(k) = \{-\ln[T(k)]\}^{1/2}. \quad (5)$$

$\Phi(k)$ is a phase function that must be an odd function to ensure that $n(x)$ is real. Making $\Phi(k)$ equal to zero is recommended when possible. However, the refractive-index profile that we obtained typically demands index values that are much too large or too small in this case. We found that it is possible to reduce the modulation of the refractive-index profile by using the following phase function:

$$\Phi(k) = \frac{\pi k}{k_{\min} + k_{\max}} - \frac{\pi}{2}\sin\left(N\pi\frac{k - k_{\min}}{k_{\max} - k_{\min}}\right), \quad (6)$$

where N is a real number that is typically in the 1–5 range; $k_{\min}$ and $k_{\max}$ are spectrally limiting wave numbers.

However, it still may be possible to obtain better expressions for $Q(k)$ and $\Phi(k)$, since they are not exact solutions.

Applying a Fourier transformation to Eq. (1) and integrating with respect to x, Sossi and Kard derived

$$n(x) = \exp\left\{\frac{2}{\pi}\int_0^{\infty} \frac{Q(k)}{k}\sin[\Phi(k) - kx]\mathrm{d}k\right\}. \quad (7)$$

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This is the central equation that is used for calculating the refractive-index profile that corresponds to the desired spectral characteristics. An analytical integration of this expression is possible in only the simplest cases. At the same time the layer thicknesses that can be obtained are limited, e.g., because of internal stress in the thin film.

In practice the refractive-index profiles are derived by numerical integration of Eq. (7) within some chosen x range. The number of k values may differ from the number of sublayers in the numerical integration. However, for practical reasons we always work with the same resolution in both cases. With $n(x)$ expressed in descrete numbers, we evaluated the corresponding spectral performance, using conventional matrix multiplications and assuming each layer element to be homogeneous (see, e.g., Ref. 6).

Refinements by Successive Approximations

Limiting the range of x and performing approximate calculations make the spectral transmittance that we obtained differ from the desired curve. However, it is often possible to reduce the deviations that are introduced by refining the techniques through successive approximations, as discussed in Ref. 4. The method that we use can be explained as follows:

(1) The desired spectral transmittance is used for calculating the first $Q(k)$ values, which are saved as $Q_o(k)$.

(2) The resulting refractive-index profile and the transmission curve that we achieved are calculated. The transmission curve that we obtained is used for the calculation of an intermediate set of $Q(k)$ values, which are saved as $Q_a(k)$.

(3) A new set of $Q(k)$ values is calculated by

$$Q(k)_j = Q(k)_{j-1} + [Q_o(k) - Q_a(k)], \qquad (8)$$

where the j index indicates the number of calculation turns.

(4) Finally it is important to set the following control:

$$\text{if } Q(k)_j < 0, \quad \text{then} \quad Q(k)_j = 0. \qquad (9)$$

Approximations are done successively until the desired transmission curve is obtained.

Designing Filters with Steep Skirts and Highly Reflective Regions

When working with the inverse Fourier technique, it appears to be quite difficult to design filters with steep skirts and/or high reflection. Figure 1 shows an example of such a filter. Choosing a phase function as expressed in Eq. (6) ($N = 1.5$) and performing the inverse Fourier transformation, see Eq. (7), one obtains the refractive-index profile shown in Fig. 2. The corresponding transmission curve is shown in Fig. 3 (dashed curve). It is apparent that it differs somewhat from the desired spectral characteristic.

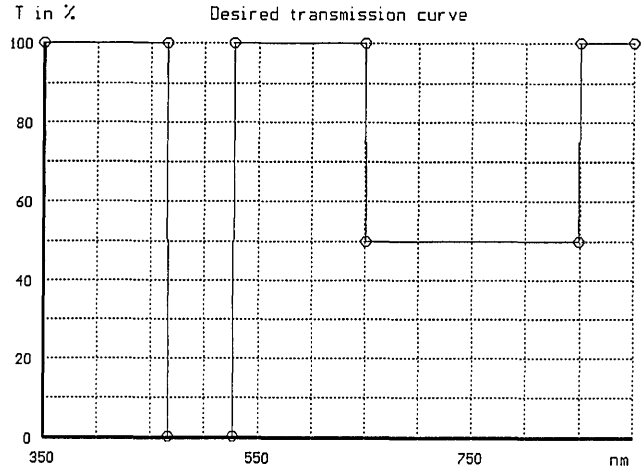

Fig. 1. Spectral characteristic for a filter with steep skirts, high reflection, and a complex function.

By performing successive approximations, it is possible to enhance the reflection in the rejection range (465–525 nm here) and to pull the transmission curve somewhat to the desired 50/50 splitting line (650–850 nm here) (see the solid curve in Fig. 3). However, it is obvious that the transmission curve obtained differs significantly from the desired characteristics in the transition regions and that it just becomes worse with successive approximations. At the same time it is not possible to lower the transmission modulations into the 650–850-nm range, and Fig. 4 shows how the refractive-index profile partially drops outside the allowed refractive-index range.

In the following we demonstrate how it is possible to reduce the observed problems significantly by taking into account the actual thickness of the thin film.

Using an Analogy to Understand the Problem

It is well-known that the technique of Fourier transformation can be used for acoustic spectrum analysis.

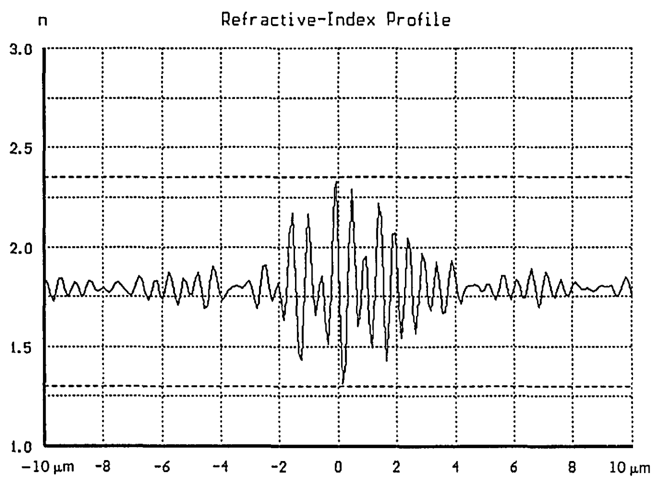

Fig. 2. Refractive-index profile obtained from calculations of the curve in Fig. 1. The total optical thickness is 10 µm, and the acceptable limits of the refractive index have been chosen to be 2.35 and 1.3.

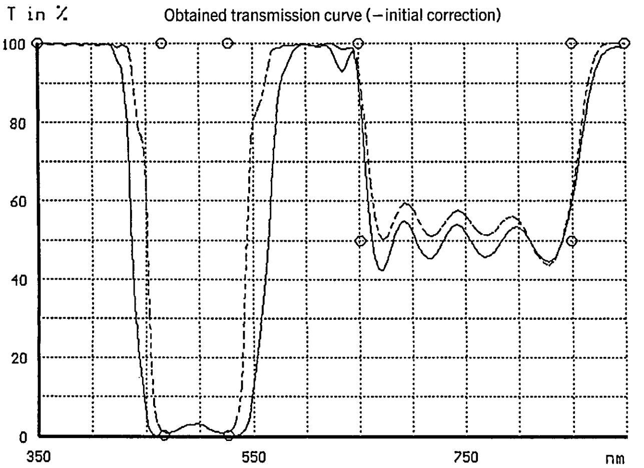

Fig. 3. Dashed curve shows the transmission corresponding to the refractive-index profile in Fig. 2. The solid curve shows the transmission curve that is obtained after three successive approximations. It is obvious that the transmission curve differs significantly from the desired characteristics in the transition regions.

It is possible to divide an acoustic signal into a sum of sine-varying tones with different amplitudes, frequencies and phases.

Since the spectral characteristic of a thick rugate structure with a limited index modulation n_p is a narrowband reflection, it seems reasonable that it should be possible to obtain a specific optical performance of a gradient-index filter (GIF) by combining sine-varying, refractive-index profiles (rugates) with differences in index modulations, propagation constants, and phases:

$$n_i = n_a + \frac{n_p}{2}\sin(k_i x - \Phi_i), \tag{10}$$

where

$$k_i = 2\pi \times n_a/\lambda_i. \tag{11}$$

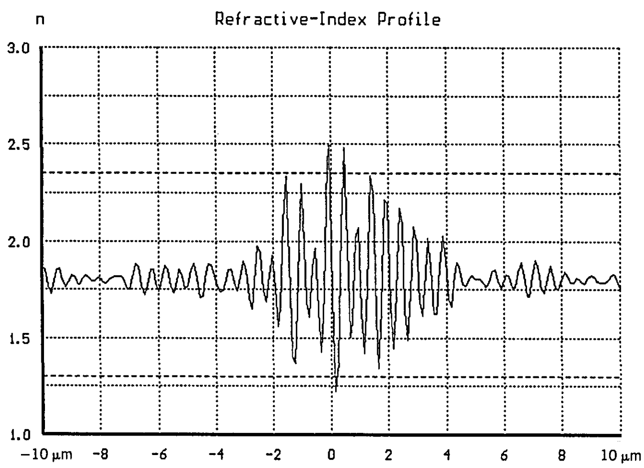

Fig. 4. Three successive approximations that have caused the refractive-index profile to drop partially outside the allowed index range.

However, in practice the thickness of a thin film is limited. This puts a limit on the number of index modulations in each rugate component. As a consequence the bandwidth of each reflection spike broadens, giving rise to the observed problems when designing filters with steep skirts and/or high reflection. Initial corrections of the input spectral characteristics, based on analytical calculations on rugate structures, are used as a basis for a new correction procedure, which makes it possible to obtain better solutions.

Description of the Developed Correction Technique

The transmission curve is calculated on the basis of M numbers describing the refractive-index profile. Each number represents the center value of the refractive index in a thin slice of the inhomogeneous layer. By calculating the transmittance curve by the matrix method,[6] each layer element is treated as if it were homogeneous with a fixed optical thickness. The number of sublayers M is 200 in this paper.

The initial correction technique that we developed can be explained as follows:

(1) Given the desired transmission curve and a fixed optical coating thickness, OT, we select the reflection values $[R(\lambda) = 1 - T(\lambda)]$ one by one with the number of wavelength samples equal to M.

(2) At each wavelength we determine the index modulation n_p that would be needed, if the actual reflection were to be obtained by means of a rugate filter (peak reflection) with a chosen optical thickness OT [see Eq. (12)].

(3) We calculate the bandwidth and the spectral characteristic of the actual rugate filter by using Eqs. (14) and (19).

(4) We controll whether the reflectivity of the rugate filter exceeds the reflectivity of the desired filter at any wavelength within the rejection band of the rugate filter. (The rejection band is determined by the bandwidth.)

(5) If the reflectivity of the rugate filter exceeds that of the desired filter within the rejection band, we calculate the amount that is necessary to traverse the examined point [see point (1)] to avoid this. Situations arise from time to time where it is not possible to move the points as needed. This is typically the case when searching for a solution with narrow-band spikes. In this case the software should warn one that the initial correction is not possible and suggest that OT be increased [see Eqs. (12) and (19)].

(6) Points are moved as needed, and the M new wavelengths are saved for future use [see point (7)]. By employing linear interpolation, it is possible to calculate transmission values that describe the corrected version of the desired transmission curve at the original sampling values of the wavelength. This corrected version of the desired transmission curve is used for the calculation of the first set of Q values $Q_o(k)$ and of the refractive-index profile $n(x)$ [see Eq. (7)].

(7) We reduce the differences between the obtained and the desired transmission curves by the use of successive approximations. According to Eq. (8) the solution becomes stable when $Q_a(k) = Q_o(k)$. This means that we have to shift the obtained transmission curve in a way that is analogous to the procedure in (6) before the calculation of the $Q_a(k)$ values.

The necessary calculations on the rugate structures are performed on the basis of analytical equations that were derived by means of coupled-wave theory by Southwell.[7] The equations that were derived for other reasons have been partially reformulated [Eqs. (12)–(19)] to fit into the actual context. By selecting some point $[\lambda_1, T(\lambda_1)]$ on the desired transmission curve, we obtain the necessary index modulation n_p:

$$n_p(\lambda_1) = \frac{2n_a\lambda_1}{\pi\, OT} \operatorname{arctanh}\{[1 - T(\lambda_1)]^{1/2}\}, \quad (12)$$

where OT is the total optical thickness of the desired GIF, $T(\lambda_1)$ is the desired transmission at λ_1, and

$$n_a = (n_H \cdot n_L)^{1/2}, \quad (13)$$

where n_H and n_L indicate the high and low limits of the refractive index.

It is now possible to calculate the reflection values for the rugate filter by means of

$$R(\lambda) = \frac{\beta^2 \sinh^2(sL)}{s^2 \cosh^2(sL) + (\alpha/2)^2 \sinh^2(sL)}, \quad (14)$$

where

$$\beta = \pi n_p/(2\lambda), \quad (15)$$

$$\alpha = 4\pi n_a(1/\lambda - 1/\lambda_1), \quad (16)$$

$$L = OT/n_a, \quad (17)$$

$$s = [\beta^2 - (\alpha/2)^2]^{1/2}, \quad (18)$$

where s is a real number as long as we remain within the bandwidth of the rugate.

The bandwidth of the rugate is defined as the width of the central reflection spike, which is the difference in wavelength between the two closest points around the central dip where the rugate is fully transmitting:

$$d\lambda/\lambda = n_p/(2n_a). \quad (19)$$

Practical Example

We can demonstrate the efficiency of the developed correcting technique by using it on the filter that was analyzed in Figs. 1–4 in the same conditions. Figure 5 shows the corrected version of the desired transmission curve. By comparing Figs. 1 and 5 it is clear that the steep skirts and high reflection call for correction.

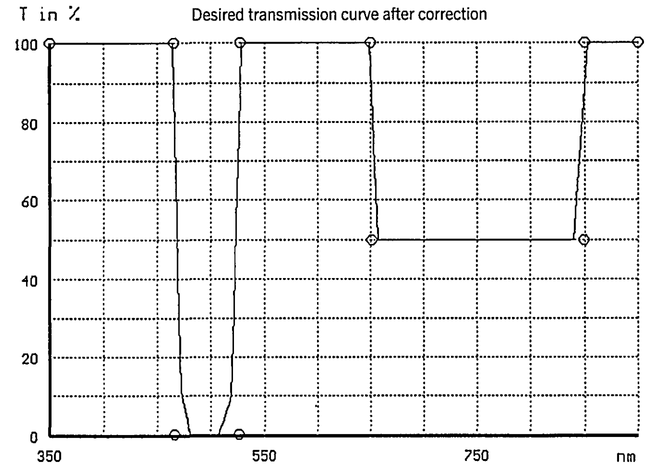

Fig. 5. Corrected version of the transmission curve in Fig. 1. By comparison it is clearly seen that the steep skirts and high reflection require correction.

The dashed curve in Fig. 6 shows the corresponding transmission curve that we obtained, which should be compared with the dashed curve in Fig. 3. It is clear how the initial correction helps to smooth and keep the transmission curve closer to what was desired.

The solid curve in Fig. 6 shows the transmission curve that we obtained as it appears after three successive approximations. By comparing it with the solid curve in Fig. 3 it is evident that the result has been improved remarkably. The ripple is in general lower, and the skirts are steeper and placed more precisely. Figure 7 shows the corresponding refractive-index profile. In Fig. 4 the refractive-index profile drops outside the allowed range. This time it remains within the range clearly showing that the new correction methods make designing easier in more than one way.

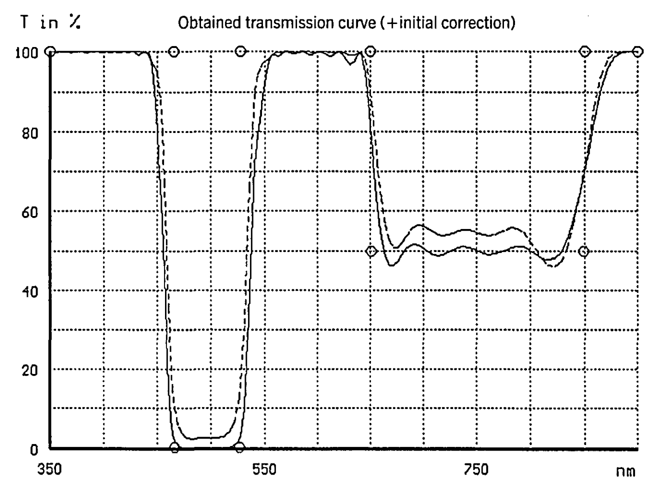

Fig. 6. Transmission curve obtained before (dashed curve) and after (solid curve) three successive approximations that correspond to Fig. 5. In comparison with Fig. 3 it is evident that the result obtained has been improved remarkably as a result of the developed initial correction method. In general the ripple is lower, and the skirts are steeper and placed more precisely.

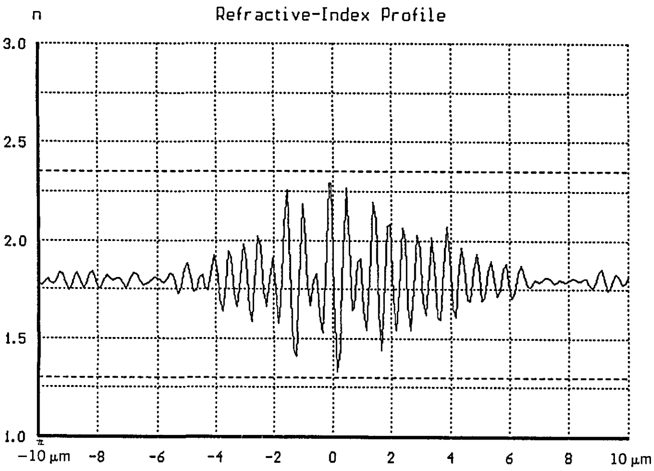

Fig. 7. Refractive-index profile that corresponds to the solid curve in Fig. 6. In Fig. 4 the refractive-index profile drops outside the allowed range. This case it remains within it, which shows clearly that the new correction methods make designing easier in more than one way.

Taking Substrates into Account

In practice a thin film must be deposited upon a carrying material or at least surrounded by homogeneous media. When substrates are included one normally introduces ripples in the transmission curve as a consequence of misadaptation between the thin film and the surroundings. Sometimes it is possible to suppress the noise by performing further successive approximations. However, this is not always possible.

Southwell has shown that it is possible in the case of rugate filters to suppress ripples caused by misadaptation between the thin film and the surroundings by adding quintic matching layers.[8] However, the rugate filter is only a fundamental type of GIF that can be designed with the inverse transformation technique. Consequently we have found that in general the matching technique can be used.

Our method can be explained as follows:

(1) One has to choose an optical thickness of the matching regions T, which has to be less than or equal to half of the total optical thickness of the layer OT.

(2) The refractive-index profile is modified according to

$$n(x) = n_s + [n(x) - n_s](6t^5 - 15t^4 + 10t^3), \quad (20)$$

where n_s is the refractive index of the surrounding media only when it is larger than the lowest value of the refractive index, which can be achieved within the thin film. Otherwise n_s is the lowest value of the refractive index that can achieved within the thin film by using the actual thin-film materials:

$$t = (x - x_{\min})/T \quad \text{for} \quad x - x_{\min} \leqq T, \quad (21)$$

$$t = (x_{\max} - x)/T \quad \text{for} \quad x_{\max} - x \leqq T, \quad (22)$$

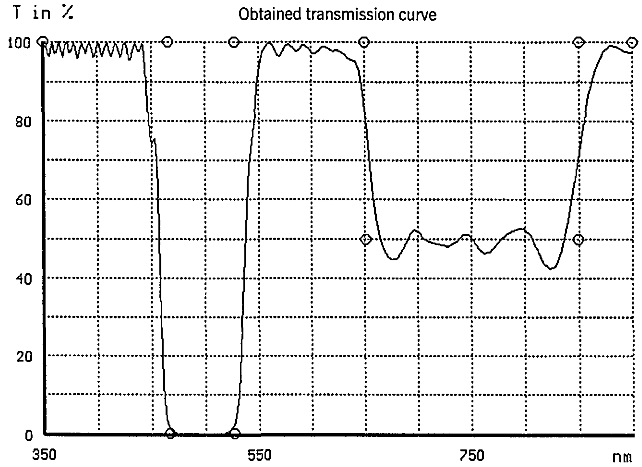

Fig. 8. Choosing surrounding media that have refractive indices in the range of, e.g., 1.5 introduces ripples caused by misadaptation at the boundaries on the transmission curve.

where $x_{\min}$ and $x_{\max}$ are the x values at the two ends of the GIF.

Note, however, that according to the work of Sossi and Kard[1-3] the carrying substrate has to be placed at the negative end of the x axis.

(3) Repeat steps (1) and (2) to find the optimal solution.

Proceeding with the Chosen Practical Example

Choosing surrounding media that have refractive indices in the range of, e.g., 1.5, introduces ripples resulting from the misadaptation at the boundaries on the transmission curve (see Fig. 8). However, by overlaying an adapting structure at each end of the refractive-index profile (see Fig. 9), it is possible to reduce the ripple (see Fig. 10).

Comparing Figs. 6 (solid curve), 8, and 10, we can see that overlaying quintic matching layers normally is an efficient way to suppress ripples caused by misadaptation between the GIF and surrounding media. However, at the same time it may also influence spectral performance in general. One ex-

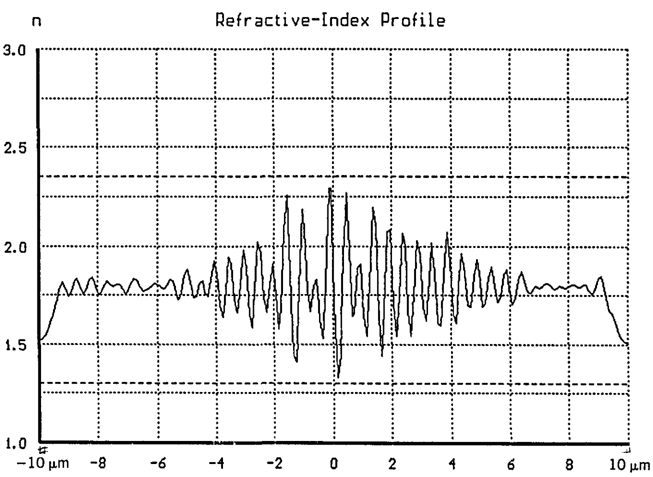

Fig. 9. Refractive-index profile when quintic matching layers are overlayed at the final 0.5 µm at each end.

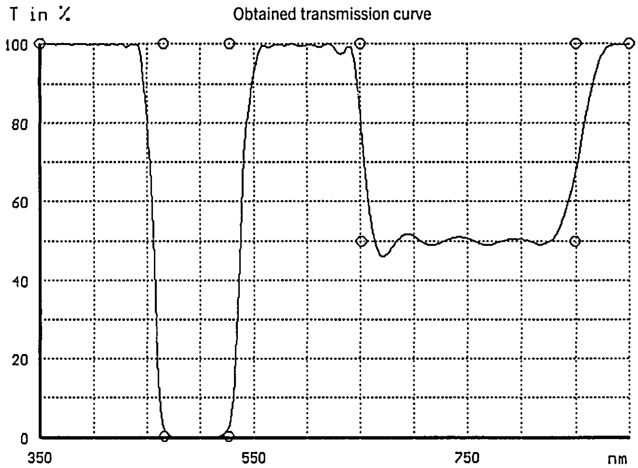

Fig. 10. Final transmission curve corresponding to the refractive-index profile in Fig. 9. It shows how it is possible to reduce the ripple by overlaying the quintic matching layers. The transmission curve should be compared with the solid curve in Fig. 6.

ample is a reduction in reflectance in highly reflecting regions. Therefore it is extremely important to realize that the successive approximation loop can be opened and closed again during different stages of the design process.

Conclusions

The refractive-index profiles obtained by performing inverse Fourier transformation often demand index values that are too large or too small. A phase function that is easy to use has been presented, which helps to reduce the modulation of the refractive-index profile. A new correction method has been demonstrated that makes it possible to obtain better solutions. This new method is of special importance when designing GIF's with steep skirts and highly reflecting regions.

Real GIF's are surrounded by homogeneous media-like glass, epoxy, and air. Normally ripples are introduced in the transmission curve as a result of misadaptation between the GIF and the surroundings. It has been shown how it is possible to suppress these ripples by overlaying quintic matching layers.

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