Achromatic prism retarder for use in polarimetric sensors

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The optical resting point of a polarimetric sensor is normally reached by using a fixed retarder plate. It is shown that this retarder plate can be omitted by using 90° prisms of selected glass materials. The sensitivity of the prism retarder toward changes in wavelength and temperature is mathematically analyzed, and it is shown how the stability of the retarder system is influenced by the selection of materials and production tolerances on prism angles. A practical example is given.

I. Introduction

Optical sensors based on the polarimetric principle have been recommended in recent years for various types of measurement, and a number of papers have been published (e.g., Refs. 1–4). Some reasons are clearly the relative simplicity and good temperature stability of this type of optical sensor compared to interferometric sensors. Published works have described the fundamentals of several commercial sensors. All the same the writer of this paper finds that it should be possible to increase the stability of polarimetric sensors by applying fundamental optical principles.

Figure 1 shows a typical sensor. Linearly polarized light is converted into circularly polarized light by means of a so-called quarterwave plate. When passing through a double refracting sensor material, the state of polarization is changed into elliptical polarization due to phase retardation. The state of the actual polarization is finally analyzed by means of a second polarizer (or two). It can easily be shown that the transmitted intensity through the whole system is given by

$$I = I_0/2 \times [1 + \cos 2v \times \cos(\delta - \tau)], \quad (1)$$

where v is the azimuth angle of the analyzer, τ is the phase retardation of the quarterwave plate,

$$\tau = (2\pi/\lambda) \times (\lambda_0/4) = \pi/2 \text{ for } \lambda = \lambda_0, \quad (2)$$

and δ is the phase retardation in the sensing medium:

$$\delta = 2\pi \times (n1 - n2) \times L/\lambda, \quad (3)$$

where L is the physical length of the sensing medium, λ is the wavelength of light in vacuum, λ_0 is the design wavelength of the quarterwave plate, and $n1$ and $n2$ are the refractive indices of the fast and slow axes in the sensing medium.

Of special interest are the situations where $v = 0°$ and $v = 90°$:

$$I(0°) = I_0/2 \times [1 + \cos(\delta - \pi/2)] = I_0/2 \times (1 + \sin\delta); \quad (4)$$

$$I(90°) = I_0/2 \times [1 - \cos(\delta - \pi/2] = I_0/2 \times (1 - \sin\delta). \quad (5)$$

In case of ac modulation it is possible to make the measurement be independent of time dependent damping in the overall system by filtering the signal electrically:

$$I_{ac}(0°)/I_{dc} = \sin(\delta) \sim \delta \text{ for } \delta \ll \pi/4. \quad (6)$$

Impulse or dc measurements can be made independent of the damping in a similar way by comparing both signals:

$$\frac{I(0°) - I(90°)}{I(0°) + I(90°)} = \sin(\delta) \sim \delta \text{ for } \delta \ll \pi/4. \quad (7)$$

However, in practice the phase retardation of the so-called quarterwave plate appears to be dependent on both wavelength and temperature.

Imagine that the phase retardation of the retarder changes somewhat ($\Delta\tau$). This results in a change in the resting point of the whole sensor system because

$$\cos(\delta - \tau) = \cos[\delta - (\pi/2 + \Delta\tau)] = \sin(\delta - \Delta\tau). \quad (8)$$

This fact can easily result in a not satisfying overall performance of the whole system. Therefore, it is of some interest to avoid the normal quarterwave retarder plate.

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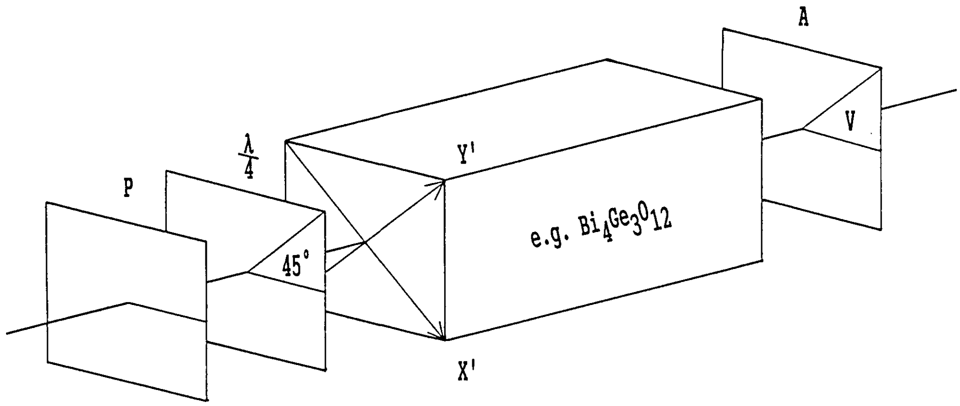

Fig. 1. Typical polarimetric sensor composed of two linear polarizers, a phase biasing retarder plate, and a double refracting sensor material.

II. Phase Retardation by Total Internal Reflection Inside a 90° Corner Prism

The fundamental idea of this paper is to use 90° prisms of glass for the fixed phase retardation. In the following it is assumed that the incident linearly polarized light is totally internally reflected in the prism as shown in Fig. 2.

If the polarization is 45° onto the plane of incidence,

$$E_{in} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \times E_o \times \sin(\omega t - ky), \tag{9}$$

the state of polarization will be changed due to the difference in phase retardation by the reflection. The vector is a directional vector of the incident electrical field as defined by Fig. 2.

If polarization parallel to the plane of incidence is indexed by p and the polarization perpendicular to the plane of incidence is indexed by s, the reflected light will be composed by the following two components:

$$E_p(\text{out}) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \times E_o/\sqrt{2} \times \sin(\omega t - kx + \delta_p), \tag{10}$$

$$E_s(\text{out}) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times E_o/\sqrt{2} \times \sin(\omega t - kx + \delta_s), \tag{11}$$

where

$$\tan(\delta_p/2) = n \times \sqrt{n^2 \times \sin^2\theta - 1}/\cos\theta, \tag{12}$$

$$\tan(\delta_s/2) = \sqrt{n^2 \times \sin^2\theta - 1}/(n \times \cos\theta). \tag{13}$$

After some calculations the following expression for the resulting phase retardation appears:

$$\delta = \delta_p - \delta_s = 2 \times \arctan[\sqrt{1 - 1/(n^2 \times \sin^2\theta)}/\tan\theta]. \tag{14}$$

One of the benefits of the prism based phase retarder is that it is nearly achromatic. This can be of some importance, e.g., when working with semiconductor light sources with device-to-device differences in wavelength and limited narrowness of linewidth (e.g., LEDs).

If the angle of incidence is set to $\theta = 45°$, the expression for the phase retardation is reduced to

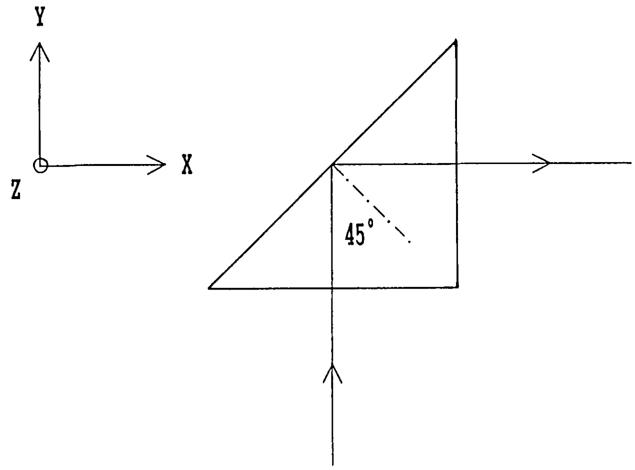

Fig. 2. If the polarization is 45° onto the plane of incidence the state of polarization will be changed due to differences in the phase retardation by the total internal reflection inside the prism.

$$\delta = 2 \times \arctan(\sqrt{1 - 2/n^2}). \tag{15}$$

Taking the derivative with respect to the wavelength of light one finds

$$\frac{\partial\delta}{\partial\lambda} = \frac{\partial\delta}{\partial n} \times \frac{\partial n}{\partial\lambda} = \frac{2}{[n(\lambda)^2 - 1] \times \sqrt{[n(\lambda)^2 - 2]}} \times \frac{\partial n}{\partial\lambda}, \tag{16}$$

which shows that the wavelength dependence of the phase retardation is reduced for larger refractive indices. It is, however, of some importance that the dispersion in the refractive index of glasses often increases with larger indices, which draws in the opposite direction. It is especially interesting to note that the dependence of wavelength is very large for refractive indices near $\sqrt{2}$.

Because of the dependence of the temperature of the refractive index of the prism the actual phase retardation depends on the temperature as given by

$$\frac{\partial\delta}{\partial T} = \frac{\partial\delta}{\partial n} \times \frac{\partial n}{\partial T} = \frac{2}{[n(T)^2 - 1] \times \sqrt{[n(T)^2 - 2]}} \times \frac{\partial n}{\partial T}, \tag{17}$$

which is similar to formula (16); thus the comments are not repeated.

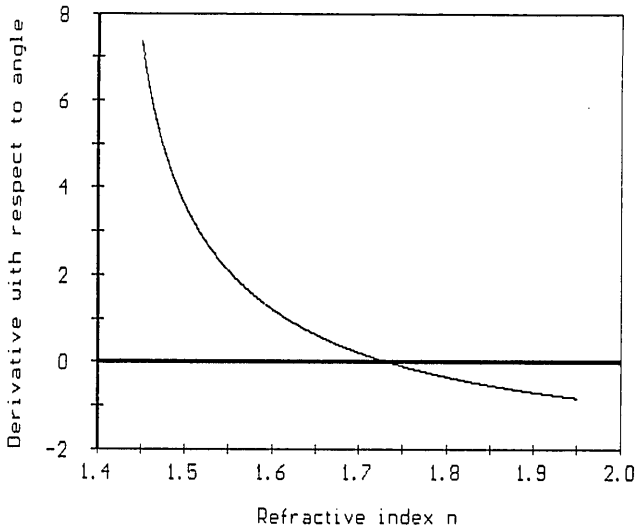

Fig. 3. Sensitivity toward manufacturing tolerances, alignment, and vibrations is high for refractive indices near $\sqrt{2}$ but very low for indices near $\sqrt{3}$.

The angle sensitivity of the phase retardation is a very important quantity, because it describes the claims on the alignment and production tolerances on the prisms as well as of the sensitivity toward vibrations.

Taking the derivative of expression (14) with respect to the angle of incidence θ leads to a somehow complex expression:

$$\frac{\partial\delta}{\partial\theta} = \frac{2}{1 - 1/(n \times \tan\theta)^2} \times \left[\frac{\cos\theta}{n \times \tan\theta^* \sqrt{(n \times \sin\theta)^2 - 1}} - \sqrt{1 - 1/(n \times \sin\theta)^2}\right]. \quad (18)$$

Choosing θ to be 45° and reducing lead to:

$$\left.\frac{\partial\delta}{\partial\theta}\right|\theta = 45° = \frac{2 \times n \times (3 - n^2)}{(n^2 - 1) \times \sqrt{(n^2 - 2)}}. \quad (19)$$

It is noted that the expression contains an interesting pole as well as an interesting root this time. From this it is seen that

(1) the sensitivity toward manufacturing tolerances, alignment, and vibrations is high for refractive indices near $n = \sqrt{2}$;

(2) the sensitivity toward the same quantities is low for refractive indices near $n = \sqrt{3}$.

Figure 3 shows a graphic expression of formula (19) in the refractive index range from 1.45 to 1.95, which is relevant for optical glasses.

III. Practical Example

Because of the limited refractive index interval it is not possible to have a phase retardation of $\pi/2$ with only one reflection, but it is not difficult to obtain a retardation of about half of it.

The total retardation of $\pi/2$ can still be reached by using one single prism if one introduces a double reflection inside a 90° prism with a refractive index of

$$n = \sqrt{2/[1 - \tan^2(\delta\text{total}/4)]} = 1.5538. \quad (20)$$

In practice it can be difficult to find a glass material with the right refractive index, especially when searching for standard glasses. Filinsky and Skettrup[5] suggest that one put some coatings on the reflecting surfaces of the prisms for correction, but this solution is not especially realistic because of the very thin layers of coating materials.

Instead of a compensating thin film coating one could use prisms of different types of glass. At a wavelength of 830 nm one could, e.g., choose to use the two standard types of glass, BaK4 and PSK3 (Schott), for the prisms. In this case one has

	BaK4 (1)	PSK3 (2)
n(830 nm)	1.56093	1.54531
$\partial n/\partial\lambda$	-2.50×10^{-5}/nm	-2.27×10^{-5}/nm
$\partial n/\partial T$	3.3×10^{-6}/K	2.5×10^{-6}/K
δ	0.800802 rad	0.766319 rad
$\partial\delta/\partial\lambda$	-5.27×10^{-5}/nm	-5.25×10^{-5}/nm
$\partial\delta/\partial T$	6.95×10^{-6}/K	5.78×10^{-6}/K
$\partial\delta/\partial\theta$	1.85351	2.18786.

Numbers can be used to estimate a worst case error in the total phase retardation of the prism system by insertion in

$$\Delta\delta_{\text{total}} < |\pi/2 - \delta 1 - \delta 2| + |\Delta\delta(\lambda)| + |\Delta\delta(T)| + |\Delta\delta(\theta)|, \quad (21)$$

where

$$\Delta\delta(\lambda) = \Delta\lambda \times (\partial\delta 1/\partial\lambda + \partial\delta 2/\partial\lambda), \quad (22)$$

$$\Delta\delta(T) = \Delta T \times (\partial\delta 1/\partial T + \partial\delta 2/\partial T), \quad (23)$$

$$\Delta\delta(\theta) = \Delta\theta \times (\partial\delta 1/\partial\theta + \partial\delta 2/\partial\theta), \quad (24)$$

and choosing the uncertainties $\Delta\lambda$, ΔT, and $\Delta\theta$ to be 20 nm, 100°, and 4′ gives a worst case value of

$$\Delta\delta_{\text{total}} \le 3.676 \text{ mrad} + 2.104 \text{ mrad} + 1.273 \text{ mrad} + 4.702 \text{ mrad},$$

$$\Delta\delta_{\text{total}} \le 11.76 \text{ mrad}.$$

Such an error of course influences the actual resting point of the sensor system. It is, however, important to notice that the error in the first line is dominated by fixed parts which depend on the choice of materials and production tolerances, and that a fixed change in retardation of this amount is of little importance because the linear approximation range of the sin function is much larger.

Optimizing the prism based quarterwave retarder for other characteristic wavelengths of semiconductor light sources, e.g., 660 and 950 nm, requires a new selection of glass materials for the prisms. It is, however, not necessary if one can accept the following error values for the BaK4 + PSK3 system:

$$\lambda = 660 \text{ nm} \Rightarrow (\pi/2 - \delta_1 - \delta_2) = 16.26 \text{ mrad};$$

$$\lambda = 950 \text{ nm} \Rightarrow (\pi/2 - \delta_1 - \delta_2) = -13.28 \text{ mrad}.$$

From this value it is clearly seen that the prism retarder in fact is nearly achromatic.

The practical configuration of the prisms can be chosen in different ways, depending on how the double

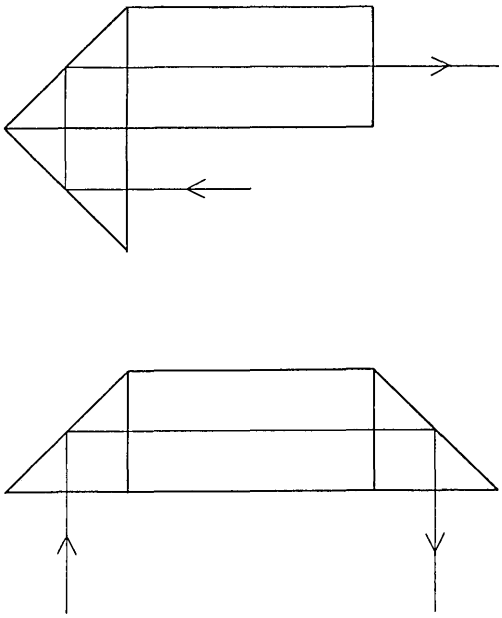

Fig. 4. Practical configuration of the prisms can be chosen in different ways depending on how the double refraction is introduced in the sensor material. Assuming a situation as shown in Fig. 1 the prisms could, e.g., be placed at each end of the sensing material. It is, however, always possible to place the two prisms side by side, in which case the light is circularly polarized when entering the sensor material.

refraction is introduced in the sensor material (see Fig. 4). Assuming a situation as shown in Fig. 1, the prisms could, e.g., be placed at each end of the sensing material. It is, however, always possible to place the two prisms side by side, in which case the light is circularly polarized when entering the sensor material.

At Light & Optics the prism based quarterwave retarder was developed especially for use with a nonlinear $Bi_4Ge_3O_{12}$ crystal, which is placed in a configuration as shown in Fig. 1. Because this electrooptical crystal has a point group symmetry of the $\bar{4}3m$ type it has neither natural birefringence, pyroelectricity, nor optical activity. For the same reason the crystal has no temperature dependence based on natural birefringence or pyroelectricity, and measurements performed on the crystal when placed in a thermoregulated chamber have shown that the retardation is very temperature stable. It is, therefore, our hope that in the end we will have a high voltage sensor for high power measurements with very good overall stability by building prisms and crystal together in one unit. Measurements in the laboratory so far have been very successful.

IV. Conclusion

Calculations have shown that it is quite easy to construct a prism based quarterwave retarder for use in polarimetric sensors when using two right angle prisms of different glass materials. Expressions have been derived which describe the derivatives of the resulting phase retardation with respect to the wavelength of light, temperature, and angle of incidence, and guiding lines for selection of glasses have been extracted.

A practical example has been given showing how to select glasses for a prism based quarterwave retarder, which is optimized for wavelengths around 830 nm, and some practical configurations of the prisms and sensor material have been proposed.

All together the presented formulas make it easy to design prism based retarders of different values for different wavelengths and to evaluate the sensitivity toward temperature and wavelength changes as well as manufacturing tolerances.

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